

PSTricks

pst-electricfield

Electric field lines of charges; v.0.13

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1 Method based on electric flux (Patrice Mégret)

Equipotential surfaces and E-field lines can be drawn by using the package `pst-func` and the command `\psplotImp[options](x1,y1)(x2,y2)`.

The following explanations describe the theory on which this is based.

Gauss theorem states that the electric flux across a closed surface S and defined by:

$$\psi = \oiint_S \vec{D} \cdot \vec{u}_n dS = Q \quad (1)$$

is equal to the real charge Q inside S . As a consequence, in place where there is no charge ($Q = 0$), the electric flux is a conservative quantity.

A tube of flux is a tube constructed on D-field lines and without charge, the flux going inside any cross-section of the tube is equal to the flux going outside any cross-section of the tube. This means that, by following a tube of a given flux, we automatically follow a D-field line. By using this technique, it is thus possible to obtain a scalar equation that describes the D-field lines. This equation is an implicit equation and can be derived for systems with simple geometrical properties.

Here the analysis will be limited to point charges and the D-field lines will thus be identical to the E-field lines as there is no electric polarization.

For a point charge q , located at the origin of the coordinate system, the electric field and the potential are given by:

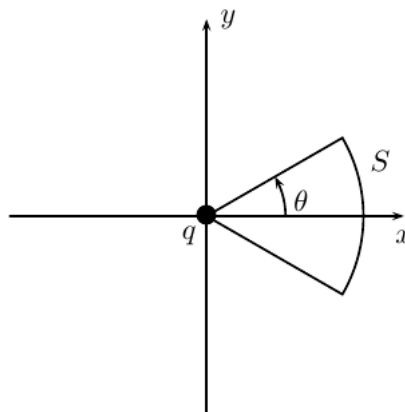
$$\vec{E} = \frac{1}{4\pi\epsilon_0\epsilon_r} q \frac{\vec{r}}{|\vec{r}|^3} \quad (2)$$

$$V = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q}{r} \quad (3)$$

The flux across a portion of a sphere of surface S and with an aperture angle θ , is simply given by:

$$\psi = \epsilon_0\epsilon_r ES = \frac{1}{2}q(1 - \cos\theta) \quad (4)$$

because $S = 2\pi r^2(1 - \cos\theta)$ and from (1) $4\pi r^2\epsilon_0\epsilon_r E = q$.



To find the implicit expression of the E-field lines, it is sufficient to express the flux

invariance:

$$\psi(x, y) = \frac{1}{2}q(1 - \cos \theta) = \text{cte} \quad (5)$$

This relation simply shows that E-field lines correspond to $\theta = \text{cte}$, so that they are clearly radial lines.

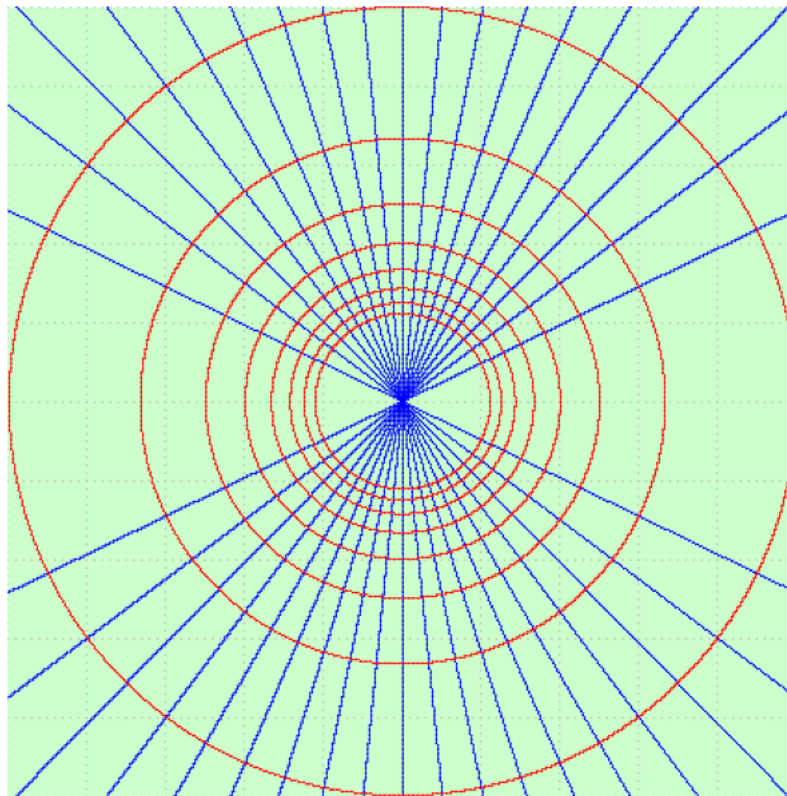
For the E-field lines in the xy plane, expression (5) in Cartesian coordinates is:

$$\frac{x}{\sqrt{x^2 + y^2}} = \text{cte} \quad (6)$$

For the equipotential surface, relation (3) is already in implicit form, therefore $V = \text{cte}$ is the wanted equation:

$$\frac{1}{\sqrt{x^2 + y^2}} = \text{cte} \quad (7)$$

The following graph shows the field and equipotential for this point charge obtained by implicit plotting of functions (6) and (7). It is clear that the E-field lines are radial ones and the equipotential surfaces cross the xy plane along circles orthogonal to the E-field lines.

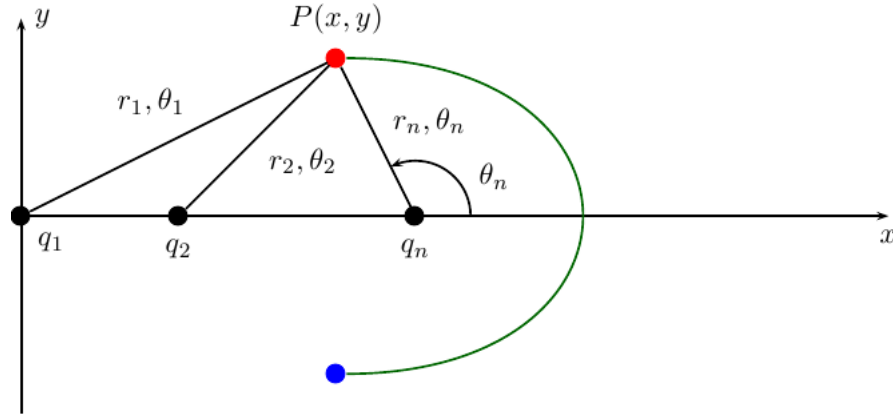


```
%% E-field lines
\multido{\r=-1+0.1}{20}{%
\psplotImp[linestyle=solid,linecolor=blue](-6,-6)(6,6){%
x y 2 exp x 2 exp add sqrt div \r \space sub}}

%% equipotential
```

```
\multido{\r=0.0+0.1}{10}{%
\psplotImp[linestyle=solid,linecolor=red](-6,-6)(6,6){%
x 2 exp y 2 exp add sqrt 1 exch div \r \space sub}}
```

Let's now generalize to point charges distributed arbitrarily along a **line**. The charge i is q_i and is placed at $(x_i, 0)$.



This problem possesses a cylindrical symmetry: it is thus sufficient to study the field and the potential in xy half-plane and the complete results are obtained by rotation around the x -axis.

By rotation around x -axis, the E-field line in P creates a tube of flux. The flux across any surface including $P(x, y)$ and crossing x -axis beyond the last charge (the trace of this surface in the xy plane is drawn in green) is obtained from (4):

$$\psi = \frac{1}{2} \sum_{i=1}^n q_i (1 - \cos \theta_i) \quad (8)$$

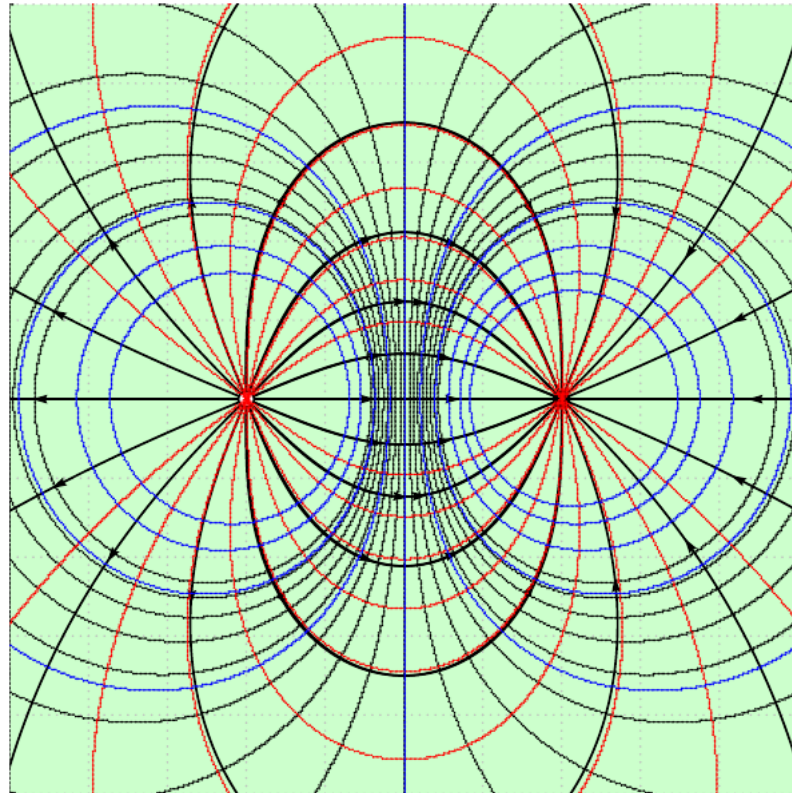
E-field lines are easily computed by the condition $\psi = \text{cte}$, which is expressed as:

$$\sum_{i=1}^n q_i \frac{x - x_i}{\sqrt{(x - x_i)^2 + y^2}} = \text{cte} \quad (9)$$

in Cartesian coordinates.

For the potential, the solution is trivial:

$$\sum_{i=1}^n \frac{q_i}{\sqrt{(x - x_i)^2 + y^2}} = \text{cte} \quad (10)$$



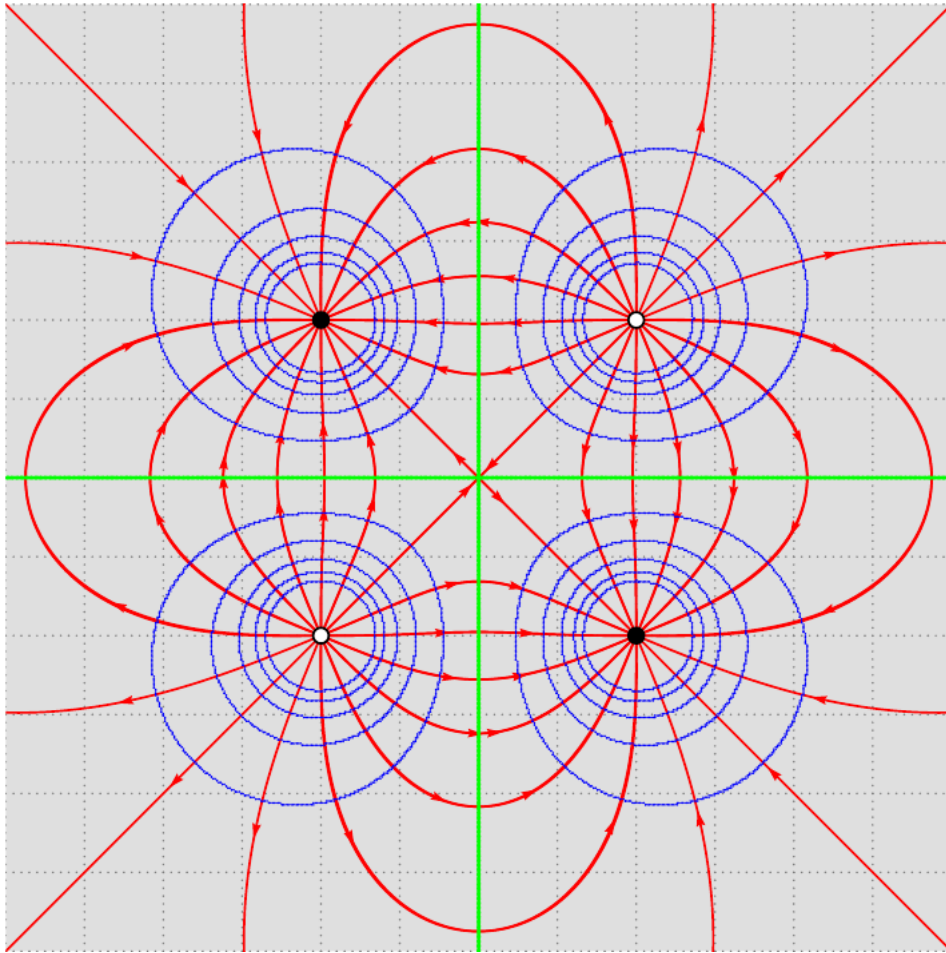
```

%% E-field lines
\multido{\r=-2+0.2}{20}{%
\psplotImp[linestyle=solid,linecolor=red](-6,-6)(6,6){%
x 2 add dup 2 exp y 2 exp add sqrt div 1 mul
x -2 add dup 2 exp y 2 exp add sqrt div -1 mul add
\r \space sub}}
%% equipotential
\multido{\r=-0.5+0.1}{10}{%
\psplotImp[linestyle=solid,linecolor=blue](-6,-6)(6,6){%
x 2 add 2 exp y 2 exp add sqrt 1 exch div 1 mul
x -2 add 2 exp y 2 exp add sqrt 1 exch div -1 mul add
\r \space sub}}

```

The last example corresponds to one charge $+1$ in $(-2, 0)$ and one charge -1 in $(2, 0)$. Here we have superposed the results obtained by implicit functions and those obtained by the direct integration of the equations. The superposition is perfect, but the method of implicit function is quite slow. Moreover, this method is limited to problem with cylindrical symmetry.

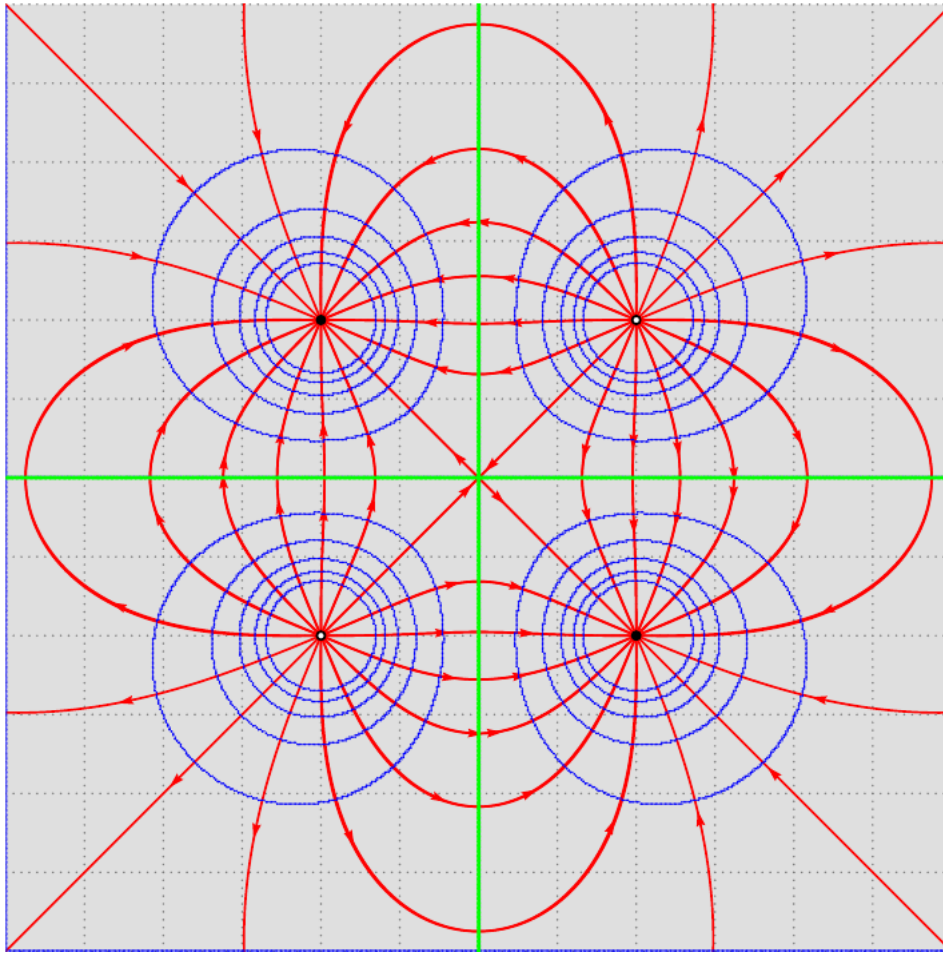
2 Examples



```

1\begin{pspicture*}(-6,-6)(6,6)
2\psframe*[linecolor=lightgray!50](-6,-6)(6,6)
3\psgrid[subgriddiv=0,gridcolor=gray,griddots=10]
4\psElectricfield[Q={[-1 -2 2][1 2 2][-1 2 -2][1 -2 -2]},linecolor=red]
5\psEquipotential[Q={[-1 -2 2][1 2 2][-1 2 -2][1 -2 -2]},linecolor=blue](-6.1,-6.1)
   (6.1,6.1)
6\psEquipotential[Q={[-1 -2 2][1 2 2][-1 2 -2][1 -2 -2]},linecolor=green,linewidth=2\
   pslinewidth,Vmax=0,Vmin=0](-6.1,-6.1)(6.1,6.1)
7\end{pspicture*}

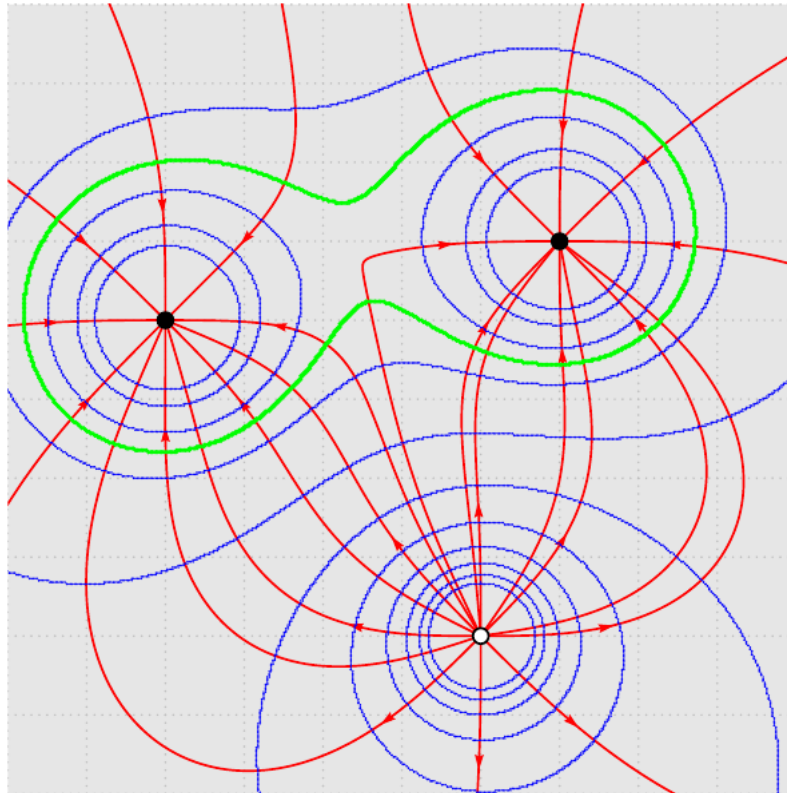
```



```

1\begin{pspicture*}(-6,-6)(6,6)
2\psframe*[linecolor=lightgray!50](-6,-6)(6,6)
3\psgrid[subgriddiv=0,gridcolor=gray,griddots=10]
4\psElectricfield[Q={[-1 -2 2 false][1 2 2 false][-1 2 -2 false][1 -2 -2 false]},radius
   =1.5pt,linecolor=red]
5\psEquipotential[Q={[-1 -2 2][1 2 2][-1 2 -2][1 -2 -2]},linecolor=blue](-6,-6)(6,6)
6\psEquipotential[Q={[-1 -2 2][1 2 2][-1 2 -2][1 -2 -2]},linecolor=green,linewidth=2\
   pslinewidth,Vmax=0,Vmin=0](-6.1,-6.1)(6.1,6.1)
7\end{pspicture*}

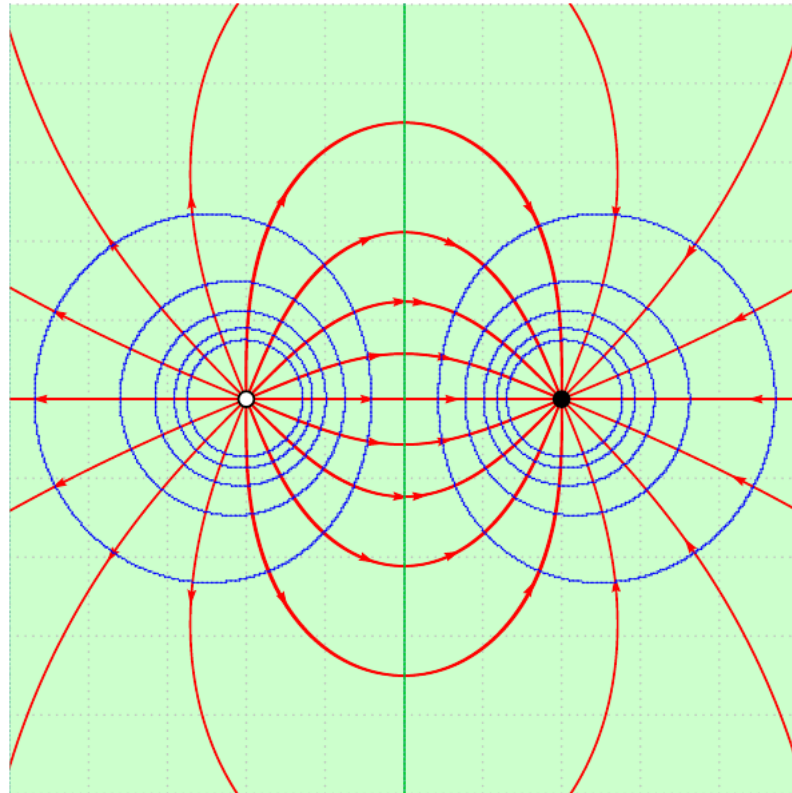
```



```

1\begin{pspicture*}(-5,-5)(5,5)
2\psframe*[linecolor=lightgray!40](-5,-5)(5,5)
3\psgrid[subgriddiv=0,gridcolor=lightgray,griddots=10]
4\psElectricfield[Q={[-1 -3 1][1 1 -3][-1 2 2]},N=9,linecolor=red,points=1000,posArrow
   =0.1,Pos=0.015]
5\psEquipotential[Q={[-1 -3 1][1 1 -3][-1 2 2]},linecolor=blue](-6,-6)(6,6)
6\psEquipotential[Q={[-1 -3 1][1 1 -3][-1 2 2]},linecolor=green,Vmin=-5,Vmax=-5,
   linewidth=2\pslinewidth](-6,-6)(6,6)
7\end{pspicture*}

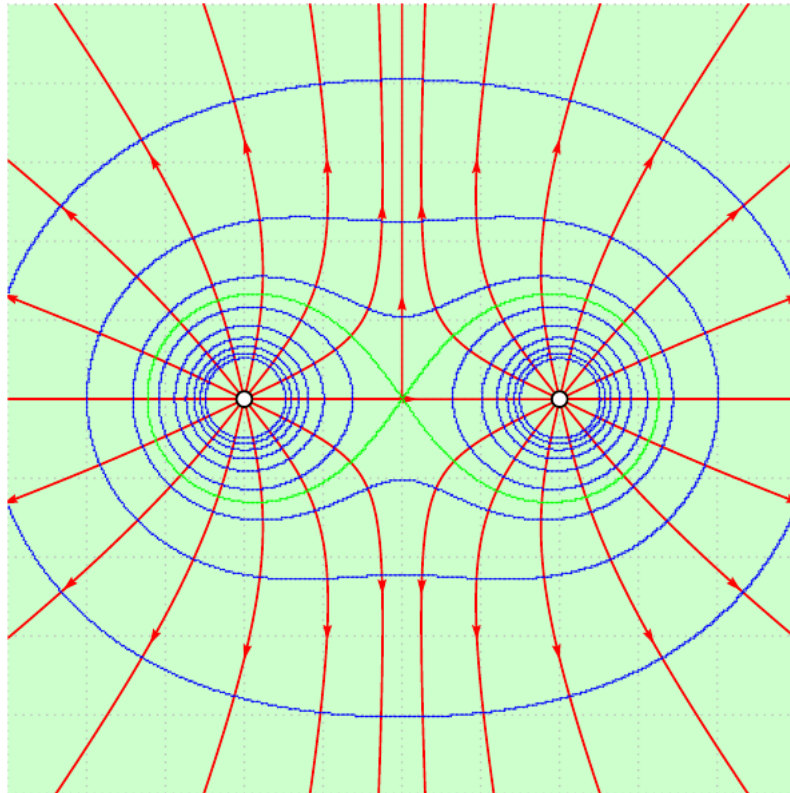
```

```

1\begin{pspicture*}(-5,-5)(5,5)
2\psframe*[linecolor=green!20](-5,-5)(5,5)
3\psgrid[subgriddiv=0,gridcolor=lightgray,griddots=10]
4\psElectricfield[Q={[1 -2 0][-1 2 0]},linecolor=red]
5\psEquipotential[Q={[1 -2 0][-1 2 0]},linecolor=blue](-5,-5)(5,5)
6\psEquipotential[Q={[1 -2 0][-1 2 0]},linecolor=green,Vmin=0,Vmax=0](-5,-5)(5,5)
7\end{pspicture*}

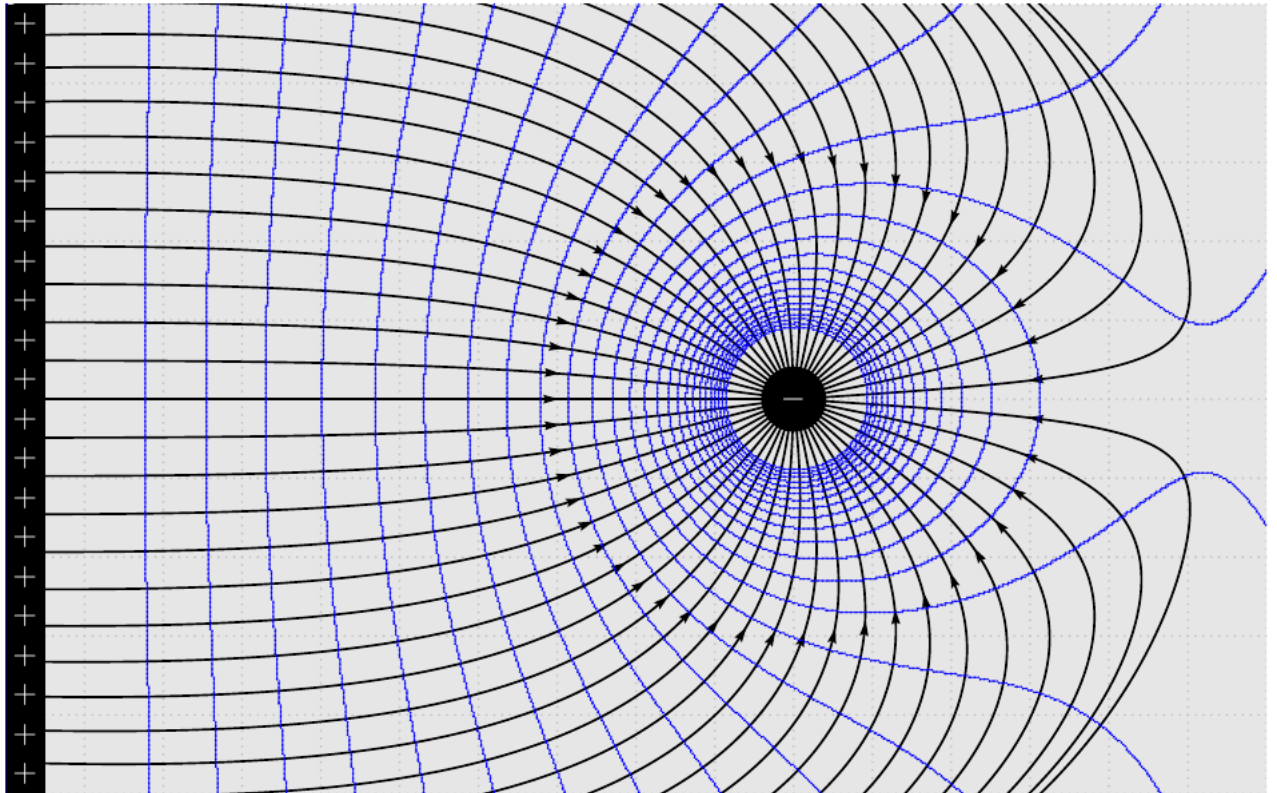
```



```

1\begin{pspicture*}(-5,-5)(5,5)
2\psframe*[linecolor=green!20](-5,-5)(5,5)
3\psgrid[subgriddiv=0,gridcolor=lightgray,griddots=10]
4\psElectricfield[Q={{[1 -2 0][1 2 0]}},linecolor=red,N=15,points=500]
5\psEquipotential[Q={{[1 -2 0][1 2 0]}},linecolor=blue,Vmin=0,Vmax=20,stepV=2](-5,-5)
6\psEquipotential[Q={{[1 -2 0][1 2 0]}},linecolor=green,Vmin=9,Vmax=9](-5,-5)(5,5)
7\end{pspicture*}

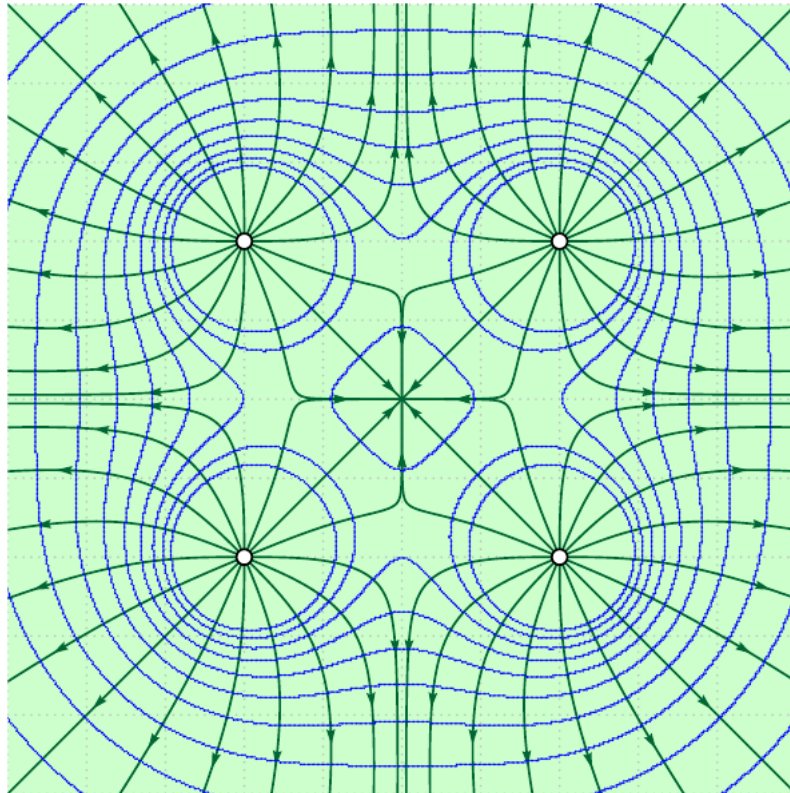
```



```

1\begin{pspicture*}(-10,-5)(6,5)
2\psframe*[linecolor=lightgray!40](-10,-5)(6,5)
3\psgrid[subgriddiv=0,gridcolor=lightgray,griddots=10]
4\psElectricfield[Q={[600 -60 0 false] [-4 0 0] },N=50,points=500,runit=0.8]
5\psEquipotential[Q={[600 -60 0 false] [-4 0 0] },linecolor=blue,Vmax=100,Vmin=50,stepV
  =2](-10,-5)(6,5)
6\psframe*(-10,-5)(-9.5,5)
7\rput(0,0){\textcolor{white}{\large$-$}}
8\multido{\rA=4.75+-0.5}{20}{\rput(-9.75,\rA){\textcolor{white}{\large$+$}}}
9\end{pspicture*}

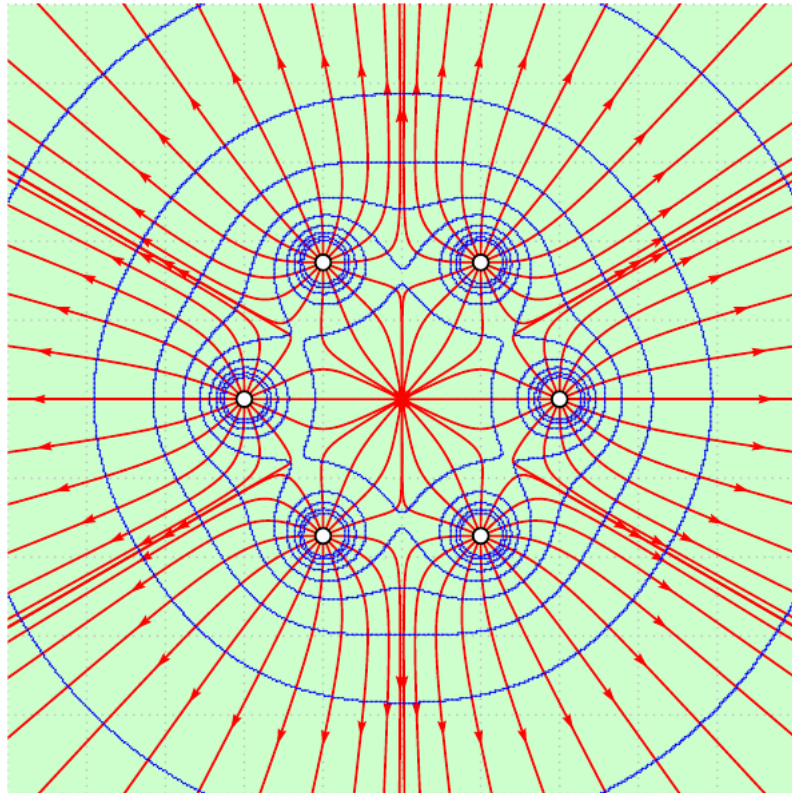
```



```

1\begin{pspicture*}(-5,-5)(5,5)
2\psframe*[linecolor=green!20](-6,-5)(6,5)
3\psgrid[subgriddiv=0,gridcolor=lightgray,griddots=10]
4\psElectricfield[Q={[1 -2 -2][1 -2 2][1 2 2][1 2 -2]},linecolor={{HTML}{006633}}]
5\psEquipotential[Q={[1 -2 -2][1 -2 2][1 2 2][1 2 -2]},Vmax=15,Vmin=0,stepV=1,linecolor
  =blue](-6,-6)(6,6)
6\end{pspicture*}

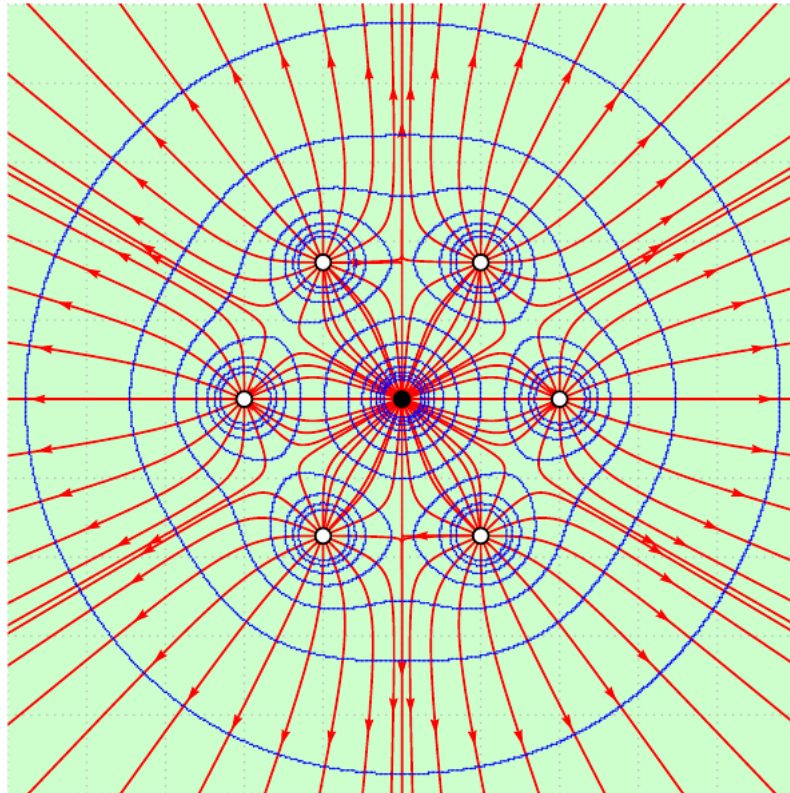
```



```

1\begin{pspicture*}(-5,-5)(5,5)
2\psframe*[linecolor=green!20](-5,-5)(5,5)
3\psgrid[subgriddiv=0,gridcolor=lightgray,griddots=10]
4\psElectricfield[Q={[1 2 0][1 1 1.732][1 -1 1.732][1 -2 0][1 -1 -1.732][1 1 -1.732]},
  linecolor=red]
5\psEquipotential[Q={[1 2 0][1 1 1.732 12][1 -1 1.732][1 -2 0][1 -1 -1.732][1 1
  -1.732]},linecolor=blue,Vmax=50,Vmin=0,stepV=5](-5,-5)(5,5)
6\end{pspicture*}

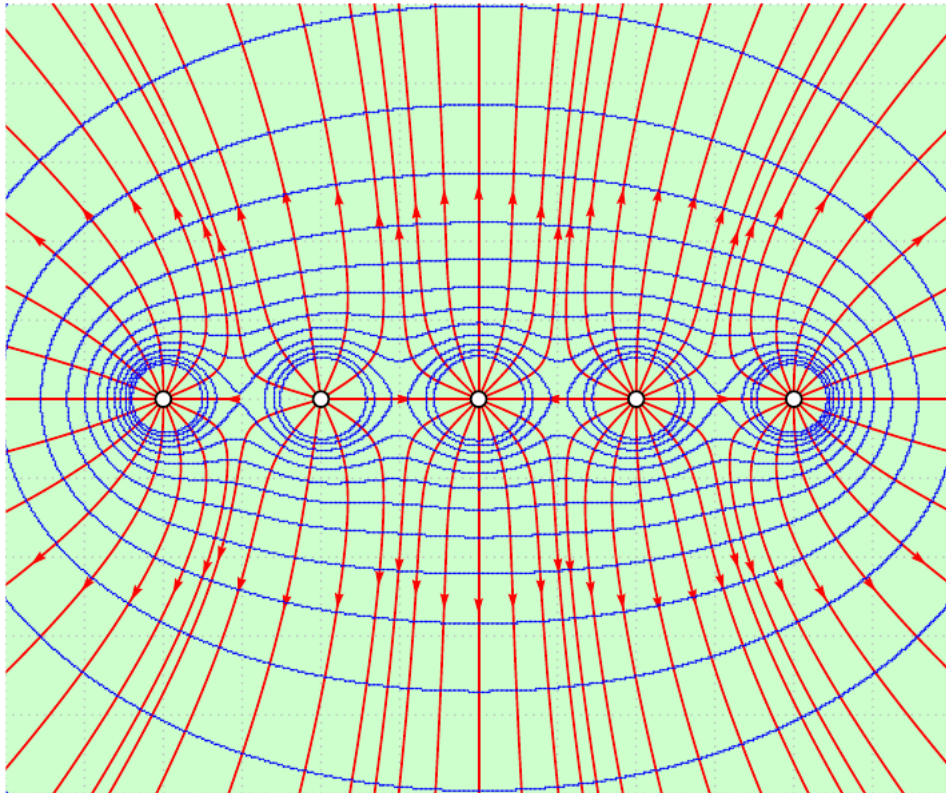
```



```

1\begin{pspicture*}(-5,-5)(5,5)
2\psframe*[linecolor=green!20](-5,-5)(5,5)
3\psgrid[subgriddiv=0,gridcolor=lightgray,griddots=10]
4\psElectricfield[Q={{[1 2 0][1 1 1.732][1 -1 1.732][1 -2 0][1 -1 -1.732][1 1 -1.732][-1
  0 0]}},linecolor=red]
5\psEquipotential[Q={{[1 2 0][1 1 1.732 12][1 -1 1.732][1 -2 0][1 -1 -1.732][1 1
  -1.732][-1 0 0]}},Vmax=40,Vmin=-10,stepV=5,linecolor=blue](-5,-5)(5,5)
6\end{pspicture*}

```



```

1\begin{pspicture*}(-6,-5)(6,5)
2\psframe*[linecolor=green!20](-6,-5)(6,5)
3\psgrid[subgriddiv=0,gridcolor=lightgray,griddots=10]
4\psElectricfield[Q={[1 -4 0][1 -2 0 12][1 0 0][1 2 0][1 4 0]},linecolor=red]
5\psEquipotential[Q={[1 -4 0][1 -2 0][1 0 0][1 2 0][1 4 0]},linecolor=blue,Vmax=30,Vmin
=0,stepV=2](-7,-5)(7,5)
6\end{pspicture*}

```

3 List of all optional arguments for pst-electricfield

Key	Type	Default
Q	ordinary	[1 -2 0 10][1 1 0][1 0 1 15]
N	ordinary	17
points	ordinary	400
Pas	ordinary	0.025
posArrow	ordinary	[none]
Vmax	ordinary	10
Vmin	ordinary	-10
stepV	ordinary	2
stepFactor	ordinary	0.67

References

- [1] M. Abramowitz and I. A. Stegun. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. National Bureau of Standards Applied Mathematics Series, U.S. Government Printing Office, Washington, D.C., USA, 1964. Corrections appeared in later printings up to the 10th Printing.
- [2] Denis Girou. Présentation de PSTricks. *Cahier GUTenberg*, 16:21–70, April 1994.
- [3] Michel Goosens, Frank Mittelbach, Sebastian Rahtz, Dennis Roegel, and Herbert Voß. *The L^AT_EX Graphics Companion*. Addison-Wesley Publishing Company, Reading, Mass., second edition, 2007.
- [4] Nikolai G. Kollock. *PostScript richtig eingesetzt: vom Konzept zum praktischen Einsatz*. IWT, Vaterstetten, 1989.
- [5] Dolan Thomas J. *Fusion Research, Volume III "Technology"*. Pergamon Press, 1982. Chapter 20 "Water-cooled magnets" , pages 600 ff "circular loops" – Integrating the Biot-Savart Law (in cylindrical geometry).
- [6] Herbert Voß. *PSTricks – Grafik für T_EX und L^AT_EX*. DANTE – Lehmanns, Heidelberg/Hamburg, fifth edition, 2008.
- [7] Timothy Van Zandt. *multido.tex - a loop macro, that supports fixed-point addition*. CTAN:/graphics/pstricks/generic/multido.tex, 1997.
- [8] Timothy Van Zandt and Denis Girou. Inside PSTricks. *TUGboat*, 15:239–246, September 1994.

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