Functional programming using Caml Light

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Contents

Chapter 1

Introduction

This document is a tutorial introduction to functional programming, and, more precisely, to the usage of Caml Light. It has been used to teach Caml Light[1] in different universities and is intended for beginners. It contains numerous examples and exercises, and absolute beginners should read it while sitting in front of a Caml Light toplevel loop, testing examples and variations by themselves.

After generalities about functional programming, some features specific to Caml Light are described. ML type synthesis and a simple execution model are presented in a complete example of prototyping a subset of ML.

Part I (chapters 2–6) may be skipped by users familiar with ML. Users with experience in functional programming, but unfamiliar with the ML dialects may skip the very first chapters and start at chapter 6, learning the Caml Light syntax from the examples. Part I starts with some intuition about functions and types and gives an overview of ML and other functional languages (chapter 2). Chapter 3 outlines the interaction with the Caml Light toplevel loop and its basic objects. Basic types and some of their associated primitives are presented in chapter 4. Lists (chapter 5) and user-defined types (chapter 6) are structured data allowing for the representation of complex objects and their easy creation and destructuration.

While concepts presented in part I are common (under one form or another) to many functional languages, part B (chapters 7–11) is dedicated to features specific to Caml Light: mutable data structures (chapter 7), exception handling (chapter 8), input/output (chapter 9) and streams and parsers (chapter 10) show a more imperative side of the language. Standalone programs and separate compilation (chapter 11) allow for modular programming and the creation of standalone applications. Concise examples of Caml Light features are to be found in this part.

Part C (chapters 12–16) is meant for already experienced Caml Light users willing to know more about how the Caml Light compiler synthesizes the types of expression and how compilation and evaluation proceeds. Some knowledge about first-order unification is assumed. The presentation is rather informal, and is sometimes terse (specially in the chapter about type synthesis). We prototype a small and simple functional language (called ASL): we give the complete prototype implementation, from the ASL parser to the symbolic execution of code. Lexing and parsing of ASL programs are presented in chapter 12, providing realistic usages of streams and parsers. Chapter 13 presents an untyped call-by-value semantics of ASL programs through the definition of an ASL interpreter. The encoding of recursion in untyped ASL is presented in chapter 14, showing the

[1] The "Caml Strong" version of these notes is available as an INRIA technical report [26].

expressive power of the language. The type synthesis of functional programs is demonstrated in chapter 15, using destructive unification (on first-order terms representing types) as a central tool. Chapter 16 introduces the Categorical Abstract Machine: a simple execution model for call-by-value functional programs. Although the Caml Light execution model is different from the one presented here, an intuition about the simple compilation of functional languages can be found in this chapter.

Warning: The programs and remarks (especially contained in parts B and C) might not be valid in Caml Light versions different from 0.7.

Part I

Functional programming

Chapter 2

Functional languages

Programming languages are said to be *functional* when the basic way of structuring programs is the notion of *function* and their essential control structure is *function application.* For example, the Lisp language [22], and more precisely its modern successor Scheme [31, 1], has been called functional because it possesses these two properties.

However, we want the programming notion of function to be as close as possible to the usual mathematical notion of function. In mathematics, functions are "first-class" objects: they can be arbitrarily manipulated. For example, they can be composed, and the composition function is itself a function.

In mathematics, one would present the *successor* function in the following way:

$$\begin{array}{rccc} \mathit{successor}: & \mathbf{N} & \longrightarrow & \mathbf{N} \\ & n & \longmapsto & n+1 \end{array}$$

The functional composition could be presented as:

$$\begin{array}{rccc} \circ: & (A \Rightarrow B) \times (C \Rightarrow A) & \longrightarrow & (C \Rightarrow B) \\ & (f, g) & \longmapsto & (x \longmapsto f\ (g\ x)) \end{array}$$

where $(A \Rightarrow B)$ denotes the space of functions from A to B.

We remark here the importance of:

1. the notion of *type*; a mathematical function always possesses a *domain* and a *codomain.* They will correspond to the programming notion of type.

2. lexical binding: when we wrote the mathematical definition of *successor*, we have assumed that the addition function $+$ had been previously defined, mapping a pair of natural numbers to a natural number; the meaning of the *successor* function is defined using the *meaning* of the addition: whatever $+$ denotes in the future, this *successor* function will remain the same.

3. the notion of *functional abstraction*, allowing to express the behavior of $f \circ g$ as $(x \longmapsto f\ (g\ x))$, i.e. the function which, when given some x, returns $f\ (g\ x)$.

ML dialects (cf. below) respect these notions. But they also allow non-functional programming styles, and, in this sense, they are functional but not *purely functional.*

2.1 History of functional languages

Some historical points:

- 1930: Alonzo Church developed the λ-calculus [6] as an attempt to provide a basis for mathematics. The λ-calculus is a formal theory for functionality. The three basic constructs of the λ-calculus are:
  - variable names (e.g. x, y,...);
  - application (MN if M and M are terms);
  - functional abstraction ($\lambda x.M$).

  Terms of the λ-calculus represent functions. The pure λ-calculus has been proved inconsistent as a logical theory. Some *type systems* have been added to it in order to remedy this inconsistency.
- 1958: Mac Carthy invented Lisp [22] whose programs have some similarities with terms of the λ-calculus. Lisp dialects have been recently evolving in order to be closer to modern functional languages (Scheme), but they still do not possess a type system.
- 1965: P. Landin proposed the ISWIM [18] language (for "If You See What I Mean"), which is the precursor of languages of the ML family.
- 1978: J. Backus introduced FP: a language of combinators [3] and a framework in which it is possible to reason about programs. The main particularity of FP programs is that they have no variable names.
- 1978: R. Milner proposes a language called ML [11], intended to be the *metalanguage* of the LCF proof assistant (i.e. the language used to program the search of proofs). This language is inspired by ISWIM (close to λ-calculus) and possesses an original type system.
- 1985: D. Turner proposed the Miranda [36] programming language, which uses Milner's type system but where programs are submitted to *lazy evaluation*.

Currently, the two main families of functional languages are the ML and the Miranda families.

2.2 The ML family

ML languages are based on a sugared[1] version of λ-calculus. Their evaluation regime is *call-by-value* (i.e. the argument is evaluated before being passed to a function), and they use Milner's type system.

The LCF proof system appeared in 1972 at Stanford (Stanford LCF). It has been further developed at Cambridge (Cambridge LCF) where the ML language was added to it.

From 1981 to 1986, a version of ML and its compiler was developed in a collaboration between INRIA and Cambridge by G. Cousineau, G. Huet and L. Paulson.

[1] i.e. with a more user-friendly syntax.

In 1981, L. Cardelli implemented a version of ML whose compiler generated native machine code.

In 1984, a committee of researchers from the universities of Edinburgh and Cambridge, Bell Laboratories and INRIA, headed by R. Milner worked on a new extended language called Standard ML [28]. This core language was completed by a module facility designed by D. MacQueen [23].

Since 1984, the Caml language has been under design in a collaboration between INRIA and LIENS[2]). Its first release appeared in 1987. The main implementors of Caml were Ascánder Suárez, Pierre Weis and Michel Mauny.

In 1989 appeared Standard ML of New-Jersey, developed by Andrew Appel (Princeton University) and David MacQueen (Bell Laboratories).

Caml Light is a smaller, more portable implementation of the core Caml language, developed by Xavier Leroy since 1990.

2.3 The Miranda family

All languages in this family use *lazy evaluation* (i.e. the argument of a function is evaluated if and when the function needs its value—arguments are passed unevaluated to functions). They also use Milner's type system.

Languages belonging to the Miranda family find their origin in the SASL language [34] (1976) developed by D. Turner. SASL and its successors (KRC [35] 1981, Miranda [36] 1985 and Haskell [15] 1990) use *sets of mutually recursive equations* as programs. These equations are written in a *script* (collection of declarations) and the user may evaluate expressions using values defined in this script. LML (Lazy ML) has been developed in Göteborg (Sweeden); its syntax is close to ML's syntax and it uses a fast execution model: the G-machine [16].

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Chapter 3

Basic concepts

We examine in this chapter some fundamental concepts which we will use and study in the following chapters. Some of them are specific to the interface with the Caml language (toplevel, global definitions) while others are applicable to any functional language.

3.1 Toplevel loop

The first contact with Caml is through its interactive aspect[1]. When running Caml on a computer, we enter a *toplevel loop* where Caml waits for input from the user, and then gives a response to what has been entered.

The beginning of a Caml Light session looks like this (assuming `$` is the shell prompt on the host machine):

```
$camllight
>       Caml Light version 0.6

#
```

On the PC version, the command is called `caml`.

The "`#`" character is Caml's prompt. When this character appears at the beginning of a line in an actual Caml Light session, the toplevel loop is waiting for a new toplevel phrase to be entered.

Throughout this document, the phrases starting by the `#` character represent legal input to Caml. Since this document has been pre-processed by Caml Light and these lines have been effectively given as input to a Caml Light process, the reader may reproduce by him/herself the session contained in each chapter (each chapter of the first two parts contains a different session, the third part is a single session). Lines starting with the "`>`" character are Caml Light error messages. Lines starting by another character are either Caml responses or (possibly) illegal input. The input is printed in typewriter font (`like this`), and output is written using slanted typewriter font (*like that*).

Important: Of course, when developing non-trivial programs, it is preferable to edit programs in files and then to include the files in the toplevel, instead of entering the programs directly.

[1]Caml Light implementations also possess a batch compiler usable to compile files and produce standalone applications; this will be discussed in chapter 11.

Furthermore, when debugging, it is very useful to *trace* some functions, to see with what arguments they are called, and what result they return. See the reference manual [21] for a description of these facilities.

3.2 Evaluation: from expressions to values

Let us enter an arithmetic expression and see what is Caml's response:

```
#1+2;;
- : int = 3
```

The expression "`1+2`" has been entered, followed by "`;;`" which represents the end of the current toplevel phrase. When encountering "`;;`", Caml enters the type-checking (more precisely *type synthesis*) phase, and prints the inferred type for the expression. Then, it compiles code for the expression, executes it and, finally, prints the result.

In the previous example, the result of evaluation is printed as "`3`" and the type is "`int`": the type of integers.

The process of evaluation of a Caml expression can be seen as transforming this expression until no further transformation is allowed. These transformations must preserve semantics. For example, if the expression "`1+2`" has the mathematical object 3 as semantics, then the result "`3`" must have the same semantics. The different phases of the Caml evaluation process are:

- parsing (checking the syntactic legality of input);
- type synthesis;
- compiling;
- loading;
- executing;
- printing the result of execution.

Let us consider another example: the application of the successor function to `2+3`. The expression `(function x -> x+1)` should be read as "the function that, given `x`, returns `x+1`". The juxtaposition of this expression to `(2+3)` is *function application*.

```
#(function x -> x+1) (2+3);;
- : int = 6
```

There are several ways to reduce that value before obtaining the result `6`. Here are two of them (the expression being reduced at each step is underlined):

$$
\begin{array}{cc}
\texttt{(function x -> x+1) }\underline{\texttt{(2+3)}} & \underline{\texttt{(function x -> x+1) (2+3)}} \\
\downarrow & \downarrow \\
\underline{\texttt{(function x -> x+1) 5}} & \underline{\texttt{(2+3)}}\texttt{ + 1} \\
\downarrow & \downarrow \\
\underline{\texttt{5+1}} & \underline{\texttt{5+1}} \\
\downarrow & \downarrow \\
\texttt{6} & \texttt{6}
\end{array}
$$

The transformations used by Caml during evaluation cannot be described in this chapter, since they imply knowledge about compilation of Caml programs and machine representation of Caml values. However, since the basic control structure of Caml is function application, it is quite easy to give an idea of the transformations involved in the Caml evaluation process by using the kind of rewriting we used in the last example. The evaluation of the (well-typed) application $e_1(e_2)$, where e_1 and e_2 denote arbitrary expressions, consists in the following steps:

- Evaluate e_2, obtaining its value v.
- Evaluate e_1 until it becomes a functional value. Because of the well-typing hypothesis, e_1 must represent a function from some type t_1 to some t_2, and the type of v is t_1. We will write (`function x ->` e) for the result of the evaluation of e_1. It denotes the mathematical object usually written as:

  $$\begin{array}{ll} f: & t_1 \rightarrow t_2 \\ & x \mapsto e \text{ (where, of course, } e \text{ may depend on } x\text{)} \end{array}$$

  N.B.: We do not evaluate e before we know the value of x.
- Evaluate e where v has been substituted for all occurrences of `x`. We then get the final value of the original expression. The result is of type t_2.

In the previous example, we evaluate:

- `2+3` to `5`;
- `(function x -> x+1)` to itself (it is already a function body);
- `x+1` where `5` is substituted for `x`, i.e. evaluate `5+1`, getting `6` as result.

It should be noticed that Caml uses call-by-value: arguments are evaluated before being passed to functions. The relative evaluation order of the functional expression and of the argument expression is implementation-dependent, and should not be relied upon. The Caml Light implementation evaluates arguments before functions (right-to-left evaluation), as shown above; the original Caml implementation evaluates functions before arguments (left-to-right evaluation).

3.3 Types

Types and their checking/synthesis are crucial to functional programming: they provide an indication about the correctness of programs.

The universe of Caml values is partitioned into *types*. A type represents a collection of values. For example, `int` is the type of integer numbers, and `float` is the type of floating-point numbers. Truth values belong to the `bool` type. Character strings belong to the `string` type. Types are divided into two classes:

- Basic types (`int`, `bool`, `string`, ...).

- Compound types such as functional types. For example, the type of functions from integers to integers is denoted by `int -> int`. The type of functions from boolean values to character strings is written `bool -> string`. The type of pairs whose first component is an integer and whose second component is a boolean value is written `int * bool`.

In fact, any combination of the types above (and even more!) is possible. This could be written as:

```
BasicType ::= int
            | bool
            | string

Type      ::= BasicType
            | Type -> Type
            | Type * Type
```

Once a Caml toplevel phrase has been entered in the computer, the Caml process starts analyzing it. First of all, it performs *syntax analysis*, which consists in checking whether the phrase belongs to the language or not. For example, here is a syntax error[2] (a parenthesis is missing):

```
#(function x -> x+1 (2+3);;
Toplevel input:
>(function x -> x+1 (2+3);;
>                        ^^
Syntax error.
```

The carets "^^^" underline the location where the error was detected.

The second analysis performed is *type analysis*: the system attempts to assign a type to each subexpression of the phrase, and to synthesize the type of the whole phrase. If type analysis fails (i.e. if the system is unable to assign a sensible type to the phrase), then a type error is reported and Caml waits for another input, rejecting the current phrase. Here is a type error (cannot add a number to a boolean):

```
#(function x -> x+1) (2+true);;
Toplevel input:
>(function x -> x+1) (2+true);;
>                       ^^^^
This expression has type bool,
but is used with type int.
```

The rejection of ill-typed phrases is called *strong typing*. Compilers for weakly typed languages (C, for example) would instead issue a warning message and continue their work at the risk of getting a "`Bus error`" or "`Illegal instruction`" message at run-time.

Once a sensible type has been deduced for the expression, then the evaluation process (compilation, loading and execution) may begin.

Strong typing enforces writing clear and well-structured programs. Moreover, it adds security and increases the speed of program development, since most typos and many conceptual errors are

[2]Actually, lexical analysis takes place before syntax analysis and *lexical errors* may occur, detecting for instance badly formed character constants.

trapped and signaled by the type analysis. Finally, well-typed programs do not need dynamic type tests (the addition function does not need to test whether or not its arguments are numbers), hence static type analysis allows for more efficient machine code.

3.4 Functions

The most important kind of values in functional programming are functional values. Mathematically, a function f of type $A \rightarrow B$ is a rule of correspondence which associates with each element of type A a unique member of type B.

If x denotes an element of A, then we will write $f(x)$ for the application of f to x. Parentheses are often useless (they are used only for grouping subexpressions), so we could also write $(f(x))$ as well as $(f((x)))$ or simply $f\ x$. The value of $f\ x$ is the unique element of B associated with x by the rule of correspondence for f.

The notation $f(x)$ is the one normally employed in mathematics to denote functional application. However, we shall be careful not to confuse a function with its application. We say "the function f with formal parameter x", meaning that f has been defined by:

$$f : x \mapsto e$$

or, in Caml, that the body of f is something like `(function x -> ...)`. Functions are values as other values. In particular, functions may be passed as arguments to other functions, and/or returned as result. For example, we could write:

```
#function f -> (function x -> (f(x+1) - 1));;
- : (int -> int) -> int -> int = <fun>
```

That function takes as parameter a function (let us call it `f`) and returns another function which, when given an argument (let us call it `x`), will return the predecessor of the value of the application `f(x+1)`.

The type of that function should be read as: `(int -> int) -> (int -> int)`.

3.5 Definitions

It is important to give names to values. We have already seen some named values: we called them *formal parameters*. In the expression `(function x -> x+1)`, the name `x` is a formal parameter. Its name is irrelevant: changing it into another one does not change the meaning of the expression. We could have written that function `(function y -> y+1)`.

If now we apply this function to, say, `1+2`, we will evaluate the expression `y+1` where `y` is the value of `1+2`. Naming `y` the value of `1+2` in `y+1` is written as:

```
#let y=1+2 in y+1;;
- : int = 4
```

This expression is a legal Caml phrase, and the `let` construct is indeed widely used in Caml programs.

The `let` construct introduces *local bindings of values to identifiers*. They are *local* because the scope of `y` is restricted to the expression `(y+1)`. The identifier `y` kept its previous binding (if any) after the evaluation of the "let ... in ..." construct. We can check that y has not been globally defined by trying to evaluate it:

```
#y;;
Toplevel input:
>y;;
>^
The value identifier y is unbound.
```

Local bindings using let also introduce *sharing* of (possibly time-consuming) evaluations. When evaluating "let x=e_1 in e_2", e_1 gets evaluated only once. All occurrences of x in e_2 access the *value* of e_1 which has been computed once. For example, the computation of:

```
let big = sum_of_prime_factors 35461243
in big+(2+big)-(4*(big+1));;
```

will be less expensive than:

```
  (sum_of_prime_factors 35461243)
+ (2 + (sum_of_prime_factors 35461243))
- (4 * (sum_of_prime_factors 35461243 + 1));;
```

The let construct also has a global form for toplevel declarations, as in:

```
#let successor = function x -> x+1;;
successor : int -> int = <fun>

#let square = function x -> x*x;;
square : int -> int = <fun>
```

When a value has been declared at toplevel, it is of course available during the rest of the session.

```
#square(successor 3);;
- : int = 16

#square;;
- : int -> int = <fun>
```

When declaring a functional value, there are some alternative syntaxes available. For example we could have declared the square function by:

```
#let square x = x*x;;
square : int -> int = <fun>
```

or (closer to the mathematical notation) by:

```
#let square (x) = x*x;;
square : int -> int = <fun>
```

All these definitions are equivalent.

3.6 Partial applications

A *partial application* is the application of a function to some but not all of its arguments. Consider, for example, the function `f` defined by:

```
#let f x = function y -> 2*x+y;;
f : int -> int -> int = <fun>
```

Now, the expression `f(3)` denotes the function which when given an argument `y` returns the value of `2*3+y`. The application `f(x)` is called a *partial application*, since `f` waits for two successive arguments, and is applied to only one. The value of `f(x)` is still a function.

We may thus define an addition function by:

```
#let addition x = function y -> x+y;;
addition : int -> int -> int = <fun>
```

This can also be written:

```
#let addition x y = x+y;;
addition : int -> int -> int = <fun>
```

We can then define the successor function by:

```
#let successor = addition 1;;
successor : int -> int = <fun>
```

Now, we may use our `successor` function:

```
#successor (successor 1);;
- : int = 3
```

Exercises

Exercise 3.1 *Give examples of functions with the following types:*

1. `(int -> int) -> int`
2. `int -> (int -> int)`
3. `(int -> int) -> (int -> int)`

Exercise 3.2 *We have seen that the names of formal parameters are meaningless. It is then possible to rename* `x` *by* `y` *in* `(function x -> x+x)`*. What should we do in order to rename* `x` *in* `y` *in*

```
(function x -> (function y -> x+y))
```

Hint: rename `y` *by* `z` *first. Question: why?*

Exercise 3.3 *Evaluate the following expressions (rewrite them until no longer possible):*

```
let x=1+2 in ((function y -> y+x) x);;
let x=1+2 in ((function x -> x+x) x);;
let f1 = function f2 -> (function x -> f2 x)
in let g = function x -> x+1
   in f1 g 2;;
```

Chapter 4

Basic types

We examine in this chapter the Caml basic types.

4.1 Numbers

Caml Light provides two numeric types: integers (type `int`) and floating-point numbers (type `float`). Integers are limited to the range $-2^{30} \ldots 2^{30}-1$. Integer arithmetic is taken modulo 2^{31}; that is, an integer operation that overflows does not raise an error, but the result simply wraps around. Predefined operations (functions) on integers include:

`+`	addition
`-`	subtraction and unary minus
`*`	multiplication
`/`	division
`mod`	modulo

Some examples of expressions:

```
#3 * 4 + 2;;
- : int = 14

#3 * (4 + 2);;
- : int = 18

#3 - 7 - 2;;
- : int = -6
```

There are precedence rules that make `*` bind tighter than `+`, and so on. In doubt, write extra parentheses.

So far, we have not seen the type of these arithmetic operations. They all expect two integer arguments; so, they are functions taking one integer and returning a function from integers to integers. The (functional) value of such infix identifiers may be obtained by taking their *prefix* version.

```
#prefix + ;;
- : int -> int -> int = <fun>
```

```
#prefix * ;;
- : int -> int -> int = <fun>

#prefix mod ;;
- : int -> int -> int = <fun>
```

As shown by their types, the infix operators +, *, ..., do not work on floating-point values. A separate set of floating-point arithmetic operations is provided:

`+.`	addition
`-.`	subtraction and unary minus
`*.`	multiplication
`/.`	division
`sqrt`	square root
`exp`, `log`	exponentiation and logarithm
`sin`, `cos`,...	usual trigonometric operations

We have two conversion functions to go back and forth between integers and floating-point numbers: `int_of_float` and `float_of_int`.

```
#1 + 2.3;;
Toplevel input:
>1 + 2.3;;
>    ^^^
This expression has type float,
but is used with type int.

#float_of_int 1 +. 2.3;;
- : float = 3.3
```

Let us now give some examples of numerical functions. We start by defining some very simple functions on numbers:

```
#let square x = x *. x;;
square : float -> float = <fun>

#square 2.0;;
- : float = 4.0

#square (2.0 /. 3.0);;
- : float = 0.444444444444

#let sum_of_squares (x,y) = square(x) +. square(y);;
sum_of_squares : float * float -> float = <fun>

#let half_pi = 3.14159 /. 2.0
#in sum_of_squares(cos(half_pi), sin(half_pi));;
- : float = 1.0
```

We now develop a classical example: the computation of the root of a function by Newton's method. Newton's method can be stated as follows: if y is an approximation to a root of a function f, then:

$$y - \frac{f(y)}{f'(y)}$$

is a better approximation, where $f'(y)$ is the derivative of f evaluated at y. For example, with $f(x) = x^2 - a$, we have $f'(x) = 2x$, and therefore:

$$y - \frac{f(y)}{f'(y)} = y - \frac{y^2 - a}{2y} = \frac{y + \frac{a}{y}}{2}$$

We can define a function `deriv` for approximating the derivative of a function at a given point by:

```
#let deriv f x dx = (f(x+.dx) -. f(x)) /. dx;;
deriv : (float -> float) -> float -> float -> float = <fun>
```

Provided `dx` is sufficiently small, this gives a reasonable estimate of the derivative of f at x.

The following function returns the absolute value of its floating point number argument. It uses the "`if` ...`then` ...`else`" conditional construct.

```
#let abs x = if x >. 0.0 then x else -. x;;
abs : float -> float = <fun>
```

The main function, given below, uses three local functions. The first one, `until`, is an example of a *recursive* function: it calls itself in its body, and is defined using a `let rec` construct (`rec` shows that the definition is recursive). It takes three arguments: a predicate `p` on floats, a function `change` from floats to floats, and a float `x`. If `p(x)` is false, then `until` is called with last argument `change(x)`, otherwise, `x` is the result of the whole call. We will study recursive functions more closely later. The two other auxiliary functions, `satisfied` and `improve`, take a float as argument and deliver respectively a boolean value and a float. The function `satisfied` asks wether the image of its argument by `f` is smaller than `epsilon` or not, thus testing wether `y` is close enough to a root of `f`. The function `improve` computes the next approximation using the formula given below. The three local functions are defined using a cascade of `let` constructs. The whole function is:

```
#let newton f epsilon =
#  let rec until p change x =
#           if p(x) then x
#           else until p change (change(x)) in
#  let satisfied y = abs(f y) <. epsilon in
#  let improve y = y -. (f(y) /. (deriv f y epsilon))
#in until satisfied improve;;
newton : (float -> float) -> float -> float -> float = <fun>
```

Some examples of equation solving:

```
#let square_root x epsilon =
#          newton (function y -> y*.y -. x) epsilon x
```

```
#and cubic_root x epsilon =
#           newton (function y -> y*.y*.y -. x) epsilon x;;
square_root : float -> float -> float = <fun>
cubic_root : float -> float -> float = <fun>

#square_root 2.0 1e-5;;
- : float = 1.41421569553

#cubic_root 8.0 1e-5;;
- : float = 2.00000000131

#2.0 *. (newton cos 1e-5 1.5);;
- : float = 3.14159265359
```

4.2 Boolean values

The presence of the conditional construct implies the presence of boolean values. The type `bool` is composed of two values `true` and `false`.

```
#true;;
- : bool = true

#false;;
- : bool = false
```

The functions with results of type `bool` are often called *predicates*. Many predicates are predefined in Caml. Here are some of them:

```
#prefix <;;
- : 'a -> 'a -> bool = <fun>

#1 < 2;;
- : bool = true

#prefix <.;;
- : float -> float -> bool = <fun>

#3.14159 <. 2.718;;
- : bool = false
```

There exist also `<=`, `>`, `>=`, and similarly `<=.`, `>.`, `>=.`.

4.2.1 Equality

Equality has a special status in functional languages because of functional values. It is easy to test equality of values of base types (integers, floating-point numbers, booleans, ...):

```
#1 = 2;;
- : bool = false

#false = (1>2);;
- : bool = true
```

But it is impossible, in the general case, to decide the equality of functional values. Hence, equality stops on a run-time error when it encounters functional values.

```
#(fun x -> x) = (fun x -> x);;
Uncaught exception: Invalid_argument "equal: functional value"
```

Since equality may be used on values of any type, what is its type? Equality takes two arguments of the same type (whatever type it is) and returns a boolean value. The type of equality is a *polymorphic type*, i.e. may take several possible forms. Indeed, when testing equality of two numbers, then its type is `int -> int -> bool`, and when testing equality of strings, its type is `string -> string -> bool`.

```
#prefix =;;
- : 'a -> 'a -> bool = <fun>
```

The type of equality uses *type variables*, written `'a`, `` `b ``, etc. Any type can be substituted to type variables in order to produces specific *instances* of types. For example, substituting `int` for `'a` above produces `int -> int -> bool`, which is the type of the equality function used on integer values.

```
#1=2;;
- : bool = false

#(1,2) = (2,1);;
- : bool = false

#1 = (1,2);;
Toplevel input:
>1 = (1,2);;
>    ^^^
This expression has type int * int,
but is used with type int.
```

4.2.2 Conditional

Conditional expressions are of the form "`if` e_{test} `then` e_1 `else` e_2", where e_{test} is an expression of type `bool` and e_1, e_2 are expressions possessing the same type. As an example, the logical negation could be written:

```
#let negate a = if a then false else true;;
negate : bool -> bool = <fun>

#negate (1=2);;
- : bool = true
```

4.2.3 Logical operators

The classical logical operators are available in Caml. Disjunction and conjunction are respectively written `or` and `&`:

```
#true or false;;
- : bool = true

#(1<2) & (2>1);;
- : bool = true
```

The operators `&` and `or` are not functions. They should not be seen as regular functions, since they evaluate their second argument only if it is needed. For example, the `or` operator evaluates its second operand only if the first one evaluates to `false`. The behavior of these operators may be described as follows:

e_1 `or` e_2	is equivalent to	`if` e_1 `then true else` e_2
e_1 `&` e_2	is equivalent to	`if` e_1 `then` e_2 `else false`

Negation is predefined:

```
#not true;;
- : bool = false
```

The `not` identifier receives a special treatment from the parser: the application "`not f x`" is recognized as "`not (f x)`" while "`f g x`" is identical to "`(f g) x`". This special treatment explains why the functional value associated to `not` can be obtained only using the `prefix` keyword:

```
#prefix not;;
- : bool -> bool = <fun>
```

4.3 Strings and characters

String constants (type `string`) are written between `"` characters (double-quotes). Single-character constants (type `char`) are written between `` ` `` characters (backquotes). The most used string operation is string concatenation, denoted by the `^` character.

```
#"Hello" ^ " World!";;
- : string = "Hello World!"

#prefix ^;;
- : string -> string -> string = <fun>
```

Characters are ASCII characters:

```
#`a`;;
- : char = `a`

#`\065`;;
- : char = `A`
```

and can be built from their ASCII code as in:

```
#char_of_int 65;;
- : char = `A`
```

Other operations are available (`sub_string`, `int_of_char`, etc). See the Caml Light reference manual [21] for complete information.

4.4 Tuples

4.4.1 Constructing tuples

It is possible to combine values into tuples (pairs, triples, ...). The *value constructor* for tuples is the "`,`" character (the comma). We will often use parentheses around tuples in order to improve readability, but they are not strictly necessary.

```
#1,2;;
- : int * int = 1, 2

#1,2,3,4;;
- : int * int * int * int = 1, 2, 3, 4

#let p = (1+2, 1<2);;
p : int * bool = 3, true
```

The infix "`*`" identifier is the *type constructor* for tuples. For instance, $t_1 * t_2$ corresponds to the mathematical cartesian product of types t_1 and t_2.

We can build tuples from any values: the tuple value constructor is *generic*.

4.4.2 Extracting pair components

Projection functions are used to extract components of tuples. For pairs, we have:

```
#fst;;
- : 'a * 'b -> 'a = <fun>

#snd;;
- : 'a * 'b -> 'b = <fun>
```

They have polymorphic types, of course, since they may be applied to any kind of pair. They are predefined in the Caml system, but could be defined by the user (in fact, the cartesian product itself could be defined — see section 6.1, dedicated to user-defined product types):

```
#let first (x,y) = x
#and second (x,y) = y;;
first : 'a * 'b -> 'a = <fun>
second : 'a * 'b -> 'b = <fun>

#first p;;
- : int = 3

#second p;;
- : bool = true
```

We say that `first` is a *generic* value because it works uniformly on several kinds of arguments (provided they are pairs). We often confuse between "generic" and "polymorphic", saying that such value is polymorphic instead of generic.

4.5 Patterns and pattern-matching

Patterns and pattern-matching play an important role in ML languages. They appear in all real programs and are strongly related to types (predefined or user-defined).

A *pattern* indicates the *shape* of a value. Patterns are "values with holes". A single variable (formal parameter) is a pattern (with no shape specified: it matches any value). When a value is *matched against* a pattern (this is called *pattern-matching*), the pattern acts as a filter. We already used patterns and pattern-matching in all the functions we wrote: the function body `(function x -> ...)` does (trivial) pattern-matching. When applied to a value, the formal parameter `x` gets bound to that value. The function body `(function (x,y) -> x+y)` does also pattern-matching: when applied to a value (a pair, because of well-typing hypotheses), the `x` and `y` get bound respectively to the first and the second component of that pair.

All these pattern-matching examples were trivial ones, they did not involve any test:

- formal parameters such as `x` do not impose any particular shape to the value they are supposed to match;
- pair patterns such as `(x,y)` always match for typing reasons (cartesian product is a *product type*).

However, some types do not guarantee such a uniform shape to their values. Consider the `bool` type: an element of type `bool` is either `true` or `false`. If we impose to a value of type `bool` to have the shape of `true`, then pattern-matching may fail. Consider the following function:

```
#let f = function true -> false;;
Toplevel input:
>let f = function true -> false;;
>        ^^^^^^^^^^^^^^^^^^^^^^
Warning: this matching is not exhaustive.
f : bool -> bool = <fun>
```

The compiler warns us that the pattern-matching may fail (we did not consider the `false` case).

Thus, if we apply `f` to a value that evaluates to `true`, pattern-matching will succeed (an equality test is performed during execution).

```
#f (1<2);;
- : bool = false
```

But, if `f`'s argument evaluates to `false`, a run-time error is reported:

```
#f (1>2);;
Uncaught exception: Match_failure ("", 1346, 1368)
```

Here is a correct definition using pattern-matching:

```
#let negate = function true -> false
#                    | false -> true;;
negate : bool -> bool = <fun>
```

The pattern-matching has now two cases, separated by the "|" character. Cases are examined in turn, from top to bottom. An equivalent definition of `negate` would be:

```
#let negate = function true -> false
#                    | x -> true;;
negate : bool -> bool = <fun>
```

The second case now matches any boolean value (in fact, only `false` since `true` has been caught by the first match case). When the second case is chosen, the identifier `x` gets bound to the argument of `negate`, and could be used in the right-hand part of the match case. Since in our example we do not use the value of the argument in the right-hand part of the second match case, another equivalent definition of `negate` would be:

```
#let negate = function true -> false
#                    | _ -> true;;
negate : bool -> bool = <fun>
```

Where "_" acts as a formal paramenter (matches any value), but does not introduce any binding: it should be read as "anything else".

As an example of pattern-matching, consider the following function definition (truth value table of implication):

```
#let imply = function (true,true) -> true
#                   | (true,false) -> false
#                   | (false,true) -> true
#                   | (false,false) -> true;;
imply : bool * bool -> bool = <fun>
```

Here is another way of defining `imply`, by using variables:

```
#let imply = function (true,x) -> x
#                   | (false,x) -> true;;
imply : bool * bool -> bool = <fun>
```

Simpler, and simpler again:

```
#let imply = function (true,x) -> x
#                   | (false,_) -> true;;
imply : bool * bool -> bool = <fun>
```

```
#let imply = function (true,false) -> false
#                   | _ -> true;;
imply : bool * bool -> bool = <fun>
```

Pattern-matching is allowed on any type of value (non-trivial pattern-matching is only possible on types with *data constructors*).

For example:

```
#let is_zero = function 0 -> true | _ -> false;;
is_zero : int -> bool = <fun>

#let is_yes = function "oui" -> true
#                    | "si" -> true
#                    | "ya" -> true
#                    | "yes" -> true
#                    | _ -> false;;
is_yes : string -> bool = <fun>
```

4.6 Functions

The type constructor "`->`" is predefined and cannot be defined in ML's type system. We shall explore in this section some further aspects of functions and functional types.

4.6.1 Functional composition

Functional composition is easily definable in Caml. It is of course a polymorphic function:

```
#let compose f g = function x -> f (g (x));;
compose : ('a -> 'b) -> ('c -> 'a) -> 'c -> 'b = <fun>
```

The type of `compose` contains no more constraints than the ones appearing in the definition: in a sense, it is the *most general* type compatible with these constraints.

These constraints are:

- the codomain of `g` and the domain of `f` must be the same;
- `x` must belong to the domain of `g`;
- `compose f g x` will belong to the codomain of `f`.

Let's see `compose` at work:

```
#let succ x = x+1;;
succ : int -> int = <fun>

#compose succ list_length;;
- : '_a list -> int = <fun>

#(compose succ list_length) [1;2;3];;
- : int = 4
```

4.6.2 Currying

We can define two versions of the addition function:

```
#let plus = function (x,y) -> x+y;;
plus : int * int -> int = <fun>

#let add = function x -> (function y -> x+y);;
add : int -> int -> int = <fun>
```

These two functions differ only in their way of taking their arguments. The first one will take an argument belonging to a cartesian product, the second one will take a number and return a function. The `add` function is said to be *the curried version* of `plus` (in honor of the logician Haskell Curry).

The currying transformation may be written in Caml under the form of a higher-order function:

```
#let curry f = function x -> (function y -> f(x,y));;
curry : ('a * 'b -> 'c) -> 'a -> 'b -> 'c = <fun>
```

Its inverse function may be defined by:

```
#let uncurry f = function (x,y) -> f x y;;
uncurry : ('a -> 'b -> 'c) -> 'a * 'b -> 'c = <fun>
```

We may check the types of `curry` and `uncurry` on particular examples:

```
#uncurry (prefix +);;
- : int * int -> int = <fun>

#uncurry (prefix ^);;
- : string * string -> string = <fun>

#curry plus;;
- : int -> int -> int = <fun>
```

Exercises

Exercise 4.1 *Define functions that compute the surface area and the volume of well-known geometric objects (rectangles, circles, spheres, ...).*

Exercise 4.2 *What would happen in a language submitted to call-by-value with recursion if there was no conditional construct (*`if ... then ... else ...`*)?*

Exercise 4.3 *Define the* `factorial` *function such that:*

$$\texttt{factorial}\ n = n * (n-1) * \ldots * 2 * 1$$

Give two versions of `factorial`*: recursive and tail recursive.*

Exercise 4.4 *Define the* `fibonacci` *function that, when given a number n, returns the nth Fibonacci number, i.e. the nth term u_n of the sequence defined by:*

$$\begin{aligned} u_1 &= 1 \\ u_2 &= 1 \\ u_{n+2} &= u_{n+1} + u_n \end{aligned}$$

Exercise 4.5 *What are the types of the following expressions?*

- `uncurry compose`
- `compose curry uncurry`
- `compose uncurry curry`

Chapter 5

Lists

Lists represent an important data structure, mainly because of their success in the Lisp language. Lists in ML are *homogeneous*: a list cannot contain elements of different types. This may be annoying to new ML users, yet lists are not as fundamental as in Lisp, since ML provides a facility for introducing new types allowing the user to define more precisely the data structures needed by the program (cf. chapter 6).

5.1 Building lists

Lists are empty or non empty sequences of elements. They are built with two *value constructors*:

- `[]`, the empty list (read: *nil*);
- `::`, the non-empty list constructor (read: *cons*). It takes an element e and a list l, and builds a new list whose first element (*head*) is e and rest (*tail*) is l.

The special syntax `[`e_1`;` ...`;` e_n `]` builds the list whose elements are $e_1, \ldots, e_n$. It is equivalent to e_1 `::` (e_2 `::` ...(e_n `::` `[]`)...).

We may build lists of numbers:

```
#1::2::[];;
- : int list = [1; 2]

#[3;4;5];;
- : int list = [3; 4; 5]

#let x=2 in [1; 2; x+1; x+2];;
- : int list = [1; 2; 3; 4]
```

Lists of functions:

```
#let adds =
#  let add x y = x+y
#  in [add 1; add 2; add 3];;
adds : (int -> int) list = [<fun>; <fun>; <fun>]
```

and indeed, lists of anything desired.

We may wonder what are the types of the list (data) constructors. The empty list is a list of anything (since it has no element), it has thus the type "*list of anything*". The list constructor `::` takes an element and a list (containing elements with the same type) and returns a new list. Here again, there is no type constraint on the elements considered.

```
#[];;
- : 'a list = []

#function head -> function tail -> head::tail;;
- : 'a -> 'a list -> 'a list = <fun>
```

We see here that the `list` type is a *recursive* type. The `::` constructor receives two arguments; the second argument is itself a `list`.

5.2 Extracting elements from lists: pattern-matching

We know how to build lists; we now show how to test emptiness of lists (although the equality predicate could be used for that) and extract elements from lists (e.g. the first one). We have used pattern-matching on pairs, numbers or boolean values. The syntax of patterns also includes list patterns. (We will see that any data constructor can actually be used in a pattern.) For lists, at least two cases have to be considered (empty, non empty):

```
#let is_null = function [] -> true | _ -> false;;
is_null : 'a list -> bool = <fun>

#let head = function x::_ -> x
#                  | _ -> raise (Failure "head");;
head : 'a list -> 'a = <fun>

#let tail = function _::l -> l
#                  | _ -> raise (Failure "tail");;
tail : 'a list -> 'a list = <fun>
```

The expression `raise (Failure "head")` will produce a run-time error when evaluated. In the definition of `head` above, we have chosen to forbid taking the head of an empty list. We could have chosen `tail []` to evaluate to `[]`, but we cannot return a value for `head []` without changing the type of the `head` function.

We say that the types `list` and `bool` are *sum types*, because they are defined with several alternatives:

- a list is either `[]` or `::` of ...
- a boolean value is either `true` or `false`

Lists and booleans are typical examples of sum types. Sum types are the only types whose values need run-time tests in order to be matched by a non-variable pattern (i.e. a pattern that is not a single variable).

The cartesian product is a *product* type (only one alternative). Product types do not involve run-time tests during pattern-matching, because the type of their values suffices to indicate statically what their structure is.

5.3 Functions over lists

We will see in this section the definition of some useful functions over lists. These functions are of general interest, but are not exhaustive. Some of them are predefined in the Caml Light system. See also [9] or [37] for other examples of functions over lists.

Computation of the length of a list:

```
#let rec length = function [] -> 0
#                        | _::l -> 1 + length(l);;
length : 'a list -> int = <fun>

#length [true; false];;
- : int = 2
```

Concatenating two lists:

```
#let rec append =
#    function [], l2 -> l2
#           | (x::l1), l2 -> x::(append (l1,l2));;
append : 'a list * 'a list -> 'a list = <fun>
```

The append function is already defined in Caml, and bound to the infix identifier @.

```
#[1;2] @ [3;4];;
- : int list = [1; 2; 3; 4]
```

Reversing a list:

```
#let rec rev = function [] -> []
#                     | x::l -> (rev l) @ [x];;
rev : 'a list -> 'a list = <fun>

#rev [1;2;3];;
- : int list = [3; 2; 1]
```

The map function applies a function on all the elements of a list, and return the list of the function results. It demonstrates full functionality (it takes a function as argument), list processing, and polymorphism (note the sharing of type variables between the arguments of map in its type).

```
#let rec map f =
#   function [] -> []
#          | x::l -> (f x)::(map f l);;
map : ('a -> 'b) -> 'a list -> 'b list = <fun>

#map (function x -> x+1) [1;2;3;4;5];;
- : int list = [2; 3; 4; 5; 6]

#map length [ [1;2;3]; [4;5]; [6]; [] ];;
- : int list = [3; 2; 1; 0]
```

The following function is a list iterator. It takes a function f, a base element a and a list $[x_1;\ldots;x_n]$. It computes:

$$\texttt{it\_list}\ f\ a\ [x_1;\ldots;x_n] = f\ (\ldots(f\ (f\ a\ x_1)\ x_2)\ \ldots)x_n$$

For instance, when applied to the curried addition function, to the base element `0`, and to a list of numbers, it computes the sum of all elements in the list. The expression:

```
it_list (prefix +) 0 [1;2;3;4;5]
```

evaluates to ((((0+1)+2)+3)+4)+5

i.e. to `15`.

```
#let rec it_list f a =
#       function [] -> a
#              | x::l -> it_list f (f a x) l;;
it_list : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a = <fun>

#let sigma = it_list prefix + 0;;
sigma : int list -> int = <fun>

#sigma [1;2;3;4;5];;
- : int = 15

#it_list (prefix *) 1 [1;2;3;4;5];;
- : int = 120
```

The `it_list` function is one of the many ways to iterate over a list. For other list iteration functions, see [9].

Exercises

Exercise 5.1 *Define the* `combine` *function which, when given a pair of lists, returns a list of pairs such that:*

```
combine ([x1;...;xn],[y1;...;yn]) = [(x1,y1);...;(xn,yn)]
```

The function generates a run-time error if the argument lists do not have the same length.

Exercise 5.2 *Define a function which, when given a list, returns the list of all its sublists.*

Chapter 6

User-defined types

The user is allowed to define his/her own data types. With this facility, there is no need to encode the data structures that must be manipulated by a program into lists (as in Lisp) or into arrays (as in Fortran). Furthermore, early detection of type errors is enforced, since user-defined data types reflect precisely the needs of the algorithms.

Types are either:

- *product* types,
- or *sum* types.

We have already seen examples of both kinds of types: the `bool` and `list` types are sum types (they contain values with different shapes and are defined and matched using several alternatives). The cartesian product is an example of a product type: we know statically the shape of values belonging to cartesian products.

In this chapter, we will see how to define and use new types in Caml.

6.1 Product types

Product types are *finite labeled* products of types. They are a generalization of cartesian product. Elements of product types are called *records*.

6.1.1 Defining product types

An example: suppose we want to define a data structure containing information about individuals. We could define:

```
#let jean=("Jean",23,"Student","Paris");;
jean : string * int * string * string = "Jean", 23, "Student", "Paris"
```

and use pattern-matching to extract any particular information about the person `jean`. The problem with such usage of cartesian product is that a function `name_of` returning the name field of a value representing an individual would have the same type as the general first projection for 4-tuples (and indeed would be the same function). This type is not precise enough since it allows for the application of the function to any 4-uple, and not only to values such as `jean`.

Instead of using cartesian product, we define a `person` data type:

```
#type person =
#  {Name:string; Age:int; Job:string; City:string};;
Type person defined.
```

The type `person` is the *product* of `string`, `int`, `string` and `string`. The field names provide type information and also documentation: it is much easier to understand data structures such as `jean` above than arbitrary tuples.

We have *labels* (i.e. `Name`, ...) to refer to components of the products. The order of appearance of the products components is not relevant: labels are sufficient to uniquely identify the components. The Caml system finds a canonical order on labels to represent and print record values. The order is always the order which appeared in the definition of the type.

We may now define the individual `jean` as:

```
#let jean = {Job="Student"; City="Paris";
#            Name="Jean"; Age=23};;
jean : person = {Name = "Jean"; Age = 23; Job = "Student"; City = "Paris"}
```

This example illustrates the fact that order of labels is not relevant.

6.1.2 Extracting products components

The canonical way of extracting product components is *pattern-matching*. Pattern-matching provides a way to mention the shape of values and to give (local) names to components of values. In the following example, we name `n` the numerical value contained in the field `Age` of the argument, and we choose to forget values contained in other fields (using the `_` character).

```
#let age_of = function
#     {Age=n; Name=_; Job=_; City=_} -> n;;
age_of : person -> int = <fun>

#age_of jean;;
- : int = 23
```

It is also possible to access the value of a single field, with the `.` (dot) operator:

```
#jean.Age;;
- : int = 23

#jean.Job;;
- : string = "Student"
```

Labels always refer to the most recent type definition: when two record types possess some common labels, then these labels always refer to the most recently defined type. When using modules (see section 11.2) this problem arises for types defined in the same module. For types defined in different modules, the full name of labels (i.e. with the name of the modules prepended) disambiguates such situations.

6.1.3 Parameterized product types

It is important to be able to define parameterized types in order to define *generic* data structures. The `list` type is parameterized, and this is the reason why we may build lists of any kind of values. If we want to define the cartesian product as a Caml type, we need type parameters because we want to be able to build cartesian product of *any* pair of types.

```
#type ('a,'b) pair = {Fst:'a; Snd:'b};;
Type pair defined.
```

```
#let first x = x.Fst and second x = x.Snd;;
first : ('a, 'b) pair -> 'a = <fun>
second : ('a, 'b) pair -> 'b = <fun>
```

```
#let p={Snd=true; Fst=1+2};;
p : (int, bool) pair = {Fst = 3; Snd = true}
```

```
#first(p);;
- : int = 3
```

Warning: the `pair` type is similar to the Caml cartesian product `*`, but it is a different type.

```
#fst p;;
Toplevel input:
>fst p;;
>    ^
This expression has type (int, bool) pair,
but is used with type 'a * 'b.
```

Actually, any two type definitions produce different types. If we enter again the previous definition:

```
#type ('a,'b) pair = {Fst:'a; Snd:'b};;
Type pair defined.
```

we cannot any longer extract the `Fst` component of `x`:

```
#p.Fst;;
Toplevel input:
>p.Fst;;
>^
This expression has type (int, bool) pair,
but is used with type ('a, 'b) pair.
```

since the label `Fst` refers to the *latter* type `pair` that we defined, while `p`'s type is the *former* `pair`.

6.2 Sum types

A *sum* type is the *finite labeled* disjoint union of several types. A sum type definition defines a type as being the union of some other types.

6.2.1 Defining sum types

Example: we want to have a type called `identification` whose values can be:

- either strings (name of an individual),
- or integers (encoding of social security number as a pair of integers).

We then need a type containing *both* `int * int` and character strings. We also want to preserve static type-checking, we thus want to be able to distinguish pairs from character strings even if they are injected in order to form a single type.

Here is what we would do:

```
#type identification = Name of string
#                    | SS of int * int;;
Type identification defined.
```

The type `identification` is the labeled disjoint union of `string` and `int * int`. The labels `Name` and `SS` are *injections*. They respectively inject `int * int` and `string` into a single type `identification`.

Let us use these injections in order to build new values:

```
#let id1 = Name "Jean";;
id1 : identification = Name "Jean"

#let id2 = SS (1670728,280305);;
id2 : identification = SS (1670728, 280305)
```

Values `id1` and `id2` belong to the same type. They may for example be put into lists as in:

```
#[id1;id2];;
- : identification list = [Name "Jean"; SS (1670728, 280305)]
```

Injections may possess one argument (as in the example above), or none. The latter case corresponds[1] to *enumerated types*, as in Pascal. An example of enumerated type is:

```
#type suit = Heart
#          | Diamond
#          | Club
#          | Spade;;
Type suit defined.

#Club;;
- : suit = Club
```

The type `suit` contains only 4 distinct elements. Let us continue this example by defining a type for cards.

[1] In Caml Light, there is no implicit order on values of sum types.

```
#type card = Ace of suit
#          | King of suit
#          | Queen of suit
#          | Jack of suit
#          | Plain of suit * int;;
Type card defined.
```

The type `card` is the disjoint union of:

- `suit` under the injection `Ace`,
- `suit` under the injection `King`,
- `suit` under the injection `Queen`,
- `suit` under the injection `Jack`,
- the product of `int` and `suit` under the injection `Plain`.

Let us build a list of cards:

```
#let figures_of c = [Ace c; King c; Queen c; Jack c]
#and small_cards_of c =
#    map (function n -> Plain(c,n)) [7;8;9;10];;
figures_of : suit -> card list = <fun>
small_cards_of : suit -> card list = <fun>
```

```
#figures_of Heart;;
- : card list = [Ace Heart; King Heart; Queen Heart; Jack Heart]
```

```
#small_cards_of Spade;;
- : card list =
 [Plain (Spade, 7); Plain (Spade, 8); Plain (Spade, 9); Plain (Spade, 10)]
```

6.2.2 Extracting sum components

Of course, pattern-matching is used to extract sum components. In case of sum types, pattern-matching uses dynamic tests for this extraction. The next example defines a function `color` returning the name of the color of the suit argument:

```
#let color = function Diamond -> "red"
#                   | Heart -> "red"
#                   | _ -> "black";;
color : suit -> string = <fun>
```

Let us count the values of cards (assume we are playing “belote”):

```
#let count trump = function
#    Ace _        -> 11
#  | King _       -> 4
```

```
#  | Queen _      -> 3
#  | Jack c       -> if c = trump then 20 else 2
#  | Plain (c,10) -> 10
#  | Plain (c,9)  -> if c = trump then 14 else 0
#  | _             -> 0;;
count : suit -> card -> int = <fun>
```

6.2.3 Recursive types

Some types possess a naturally recursive structure, lists, for example. It is also the case for tree-like structures, since trees have subtrees that are trees themselves.

Let us define a type for abstract syntax trees of a simple arithmetic language[2]. An arithmetic expression will be either a numeric constant, or a variable, or the addition, multiplication, difference, or division of two expressions.

```
#type arith_expr = Const of int
#                | Var of string
#                | Plus of args
#                | Mult of args
#                | Minus of args
#                | Div of args
#and args = {Arg1:arith_expr; Arg2:arith_expr};;
Type arith_expr defined.
Type args defined.
```

The two types `arith_expr` and `args` are simultaneously defined, and `arith_expr` is recursive since its definition refers to `args` which itself refers to `arith_expr`. As an example, one could represent the abstract syntax tree of the arithmetic expression "`x+(3*y)`" as the Caml value:

```
#Plus {Arg1=Var "x";
#      Arg2=Mult{Arg1=Const 3; Arg2=Var "y"}};;
- : arith_expr =
 Plus {Arg1 = Var "x"; Arg2 = Mult {Arg1 = Const 3; Arg2 = Var "y"}}
```

The recursive definition of types may lead to types such that it is hard or impossible to build values of these types. For example:

```
#type stupid = {Next:stupid};;
Type stupid defined.
```

Elements of this type are *infinite* data structures. Essentially, the only way to construct one is:

```
#let rec stupid_value = {Next=stupid_value};;
stupid_value : stupid =
 {Next =
```

[2]Syntax trees are said to be *abstract* because they are pieces of *abstract syntax* contrasting with *concrete syntax* which is the "string" form of programs: analyzing (parsing) concrete syntax usually produces abstract syntax.

```
   {Next =
     {Next =
       {Next =
         {Next =
           {Next =
             {Next =
               {Next =
                 {Next =
                   {Next =
                     {Next = {Next = {Next = {Next = {Next = {Next = .}}}}}}}}}}
}}}}}}
```

Recursive type definitions should be *well-founded* (i.e. possess a non-recursive case, or *base case*) in order to work well with call-by-value.

6.2.4 Parameterized sum types

Sum types may also be parameterized. Here is the definition of a type isomorphic to the `list` type:

```
#type 'a sequence = Empty
#                 | Sequence of 'a * 'a sequence;;
Type sequence defined.
```

A more sophisticated example is the type of generic binary trees:

```
#type ('a,'b) btree = Leaf of 'b
#                   | Btree of ('a,'b) node
#and ('a,'b) node = {Op:'a;
#                    Son1: ('a,'b) btree;
#                    Son2: ('a,'b) btree};;
Type btree defined.
Type node defined.
```

A binary tree is either a leaf (holding values of type `'b`) or a node composed of an operator (of type `'a`) and two sons, both of them being binary trees.

Binary trees can also be used to represent abstract trees for arithmetic expressions (with only binary operators and only one kind of leaves). The abstract syntax tree `t` of "`1+2`" could be defined as:

```
#let t = Btree {Op="+"; Son1=Leaf 1; Son2=Leaf 2};;
t : (string, int) btree = Btree {Op = "+"; Son1 = Leaf 1; Son2 = Leaf 2}
```

Finally, it is time to notice that pattern-matching is not restricted to function bodies, i.e. constructs such as:

function P_1 -> E_1
  | ...
  | P_n -> E_n

but there is also a construct dedicated to pattern-matching of actual values:

$$
\begin{array}{lll}
\texttt{match } E \texttt{ with} & P_1 & \texttt{->}\ E_1 \\
\quad\quad\quad\quad\quad\texttt{|} & \ldots & \\
\quad\quad\quad\quad\quad\texttt{|} & P_n & \texttt{->}\ E_n
\end{array}
$$

which matches the value of the expression E against each of the patterns P_i, selecting the first one that matches, and giving control to the corresponding expression. For example, we can match the tree `t` previously defined by:

```
#match t with Btree{Op=x; Son1=_; Son2=_} -> x
#           | Leaf l -> "No operator";;
- : string = "+"
```

6.2.5 Data constructors and functions

One may ask: "What is the difference between a sum data constructor and a function?". At first sight, they look very similar. We assimilate constant data constructors (such as `Heart`) to constants. Similarly, in Caml Light, sum data constructors with arguments also possess a functional type:

```
#Ace;;
- : suit -> card = <fun>
```

However, a data constructor possesses particular properties that a general function does not possess, and it is interesting to understand these differences. From the mathematical point of view, a sum data constructor is known to be an *injection* while a Caml function is a general function without further information. A mathematical injection $f : A \rightarrow B$ admits an inverse function f^{-1} from its image $f(A) \subset B$ to A.

From the examples above, if we consider the `King` constructor, then:

```
#let king c = King c;;
king : suit -> card = <fun>
```

`king` is the general function associated to the `King` constructor, and:

```
#function King c -> c;;
Toplevel input:
>function King c -> c;;
>^^^^^^^^^^^^^^^^^^^^
Warning: this matching is not exhaustive.
- : card -> suit = <fun>
```

is the inverse function for `king`. It is a partial function, since pattern-matching may fail.

6.2.6 Degenerate cases: when sums meet products

What is the status of a sum type with a single case such as:

```
#type counter1 = Counter of int;;
Type counter1 defined.
```

Of course, the type `counter1` is isomorphic to `int`. The injection `function x -> Counter x` is a *total* function from `int` to `counter1`. It is thus a *bijection*.

Another way to define a type isomorphic to `int` would be:

```
#type counter2 = {Counter: int};;
Type counter2 defined.
```

The types `counter1` and `counter2` are isomorphic to `int`. They are at the same time sum and product types. Their pattern-matching does not perform any run-time test.

The possibility of defining arbitrary complex data types permits the easy manipulation of abstract syntax trees in Caml (such as the `arith_expr` type above). These abstract syntax trees are supposed to represent programs of a language (e.g. a language of arithmetic expressions). These kind of languages which are defined in Caml are called *object-languages* and Caml is said to be their *metalanguage*.

6.3 Summary

- New types can be introduced in Caml.
- Types may be *parameterized* by type variables. The syntax of type parameters is:

  ```
  <params> ::
            | <tvar>
            | ( <tvar> [, <tvar>]* )
  ```

- Types can be *recursive*.
- Product types:
  - Mathematical product of several types.
  - The construct is:

    ```
    type <params> <tname> =
                     {<Field>: <type>; ...}
    ```

    where the `<type>`s may contain type variables appearing in `<params>`.
- Sum types:
  - Mathematical disjoint union of several types.
  - The construct is:

    ```
    type <params> <tname> =
             <Injection> [of <type>] | ...
    ```

    where the `<type>`s may contain type variables appearing in `<params>`.

Exercises

Exercise 6.1 *Define a function taking as argument a binary tree and returning a pair of lists: the first one contains all operators of the tree, the second one contains all its leaves.*

Exercise 6.2 *Define a function* `map_btree` *analogous to the* `map` *function on lists. The function* `map_btree` *should take as arguments two functions* `f` *and* `g`*, and a binary tree. It should return a new binary tree whose leaves are the result of applying* `f` *to the leaves of the tree argument, and whose operators are the results of applying the* `g` *function to the operators of the argument.*

Exercise 6.3 *We can associate to the* `list` *type definition an canonical iterator in the following way. We give a functional interpretation to the data constructors of the* `list` *type.*

We change the list constructors `[]` *and* `::` *respectively into a constant* `a` *and an operator* $\oplus$ *(used as a prefix identifier), and abstract with respect to these two operators, obtaining the list iterator satisfying:*

list_it $\oplus$ a [] = a
list_it $\oplus$ a (x_1::...::x_n::[]) = $x_1 \oplus (\ldots \oplus (x_n \oplus$ a$)\ldots)$

Its Caml definition would be:

```
#let rec list_it f a =
#        function [] -> a
#               | x::l -> f x (list_it f a l);;
list_it : ('a -> 'b -> 'b) -> 'b -> 'a list -> 'b = <fun>
```

As an example, the application of `it_list` *to the functional composition and to its neutral element (the identity function), computes the composition of lists of functions (try it!).*

Define, using the same method, a canonical iterator over binary trees.

Part II

Caml Light specifics

Chapter 7

Mutable data structures

The definition of a sum or product type may be annotated to allow physical (destructive) update on data structures of that type. This is the main feature of the *imperative programming* style. Writing values into memory locations is the fundamental mechanism of imperative languages such as Pascal. The Lisp language, while mostly functional, also provides the dangerous functions `rplaca` and `rplacd` to physically modify lists. Mutable structures are required to implement many efficient algorithms. They are also very convenient to represent the current state of a state machine.

7.1 User-defined mutable data structures

Assume we want to define a type `person` as in the previous chapter. Then, it seems natural to allow a person to change his/her age, job and the city that person lives in, but *not* his/her name. We can do this by annotating some labels in the type definition of `person` by the `mutable` keyword:

```
#type person =
#    {Name: string; mutable Age: int;
#     mutable Job: string; mutable City: string};;
Type person defined.
```

We can build values of type `person` in the very same way as before:

```
#let jean =
#    {Name="Jean"; Age=23; Job="Student"; City="Paris"};;
jean : person = {Name = "Jean"; Age = 23; Job = "Student"; City = "Paris"}
```

But now, the value `jean` may be physically modified in the fields specified to be `mutable` in the definition (and *only* in these fields).

We can modify the field `Field` of an expression `<expr1>` in order to assign it the value of `<expr2>` by using the following construct:

```
<expr1>.Field <- <expr2>
```

For example; if we want `jean` to become one year older, we would write:

```
#jean.Age <- jean.Age + 1;;
- : unit = ()
```

Now, the value jean has been modified into:

```
#jean;;
- : person = {Name = "Jean"; Age = 24; Job = "Student"; City = "Paris"}
```

We may try to change the Name of jean, but we won't succeed: the typecheker will not allow us to do that.

```
#jean.Name <- "Paul";;
Toplevel input:
>jean.Name <- "Paul";;
>^^^^^^^^^^^^^^^^^^^
The label Name is not mutable.
```

It is of course possible to use such constructs in functions as in:

```
#let get_older ({Age=n; _} as p) = p.Age <- n + 1;;
get_older : person -> unit = <fun>
```

In that example, we named n the current Age of the argument, but we also named p the argument. This is an *alias* pattern: it saves us the bother of writing:

```
#let get_older p =
#    match p with {Age=n} -> p.Age <- n + 1;;
get_older : person -> unit = <fun>
```

Notice that in the two previous expressions, we did not specify all fields of the record p. Other examples would be:

```
#let move p new_city = p.City <- new_city
#and change_job p j = p.Job <- j;;
move : person -> string -> unit = <fun>
change_job : person -> string -> unit = <fun>
```

```
#change_job jean "Teacher"; move jean "Cannes";
#get_older jean; jean;;
- : person = {Name = "Jean"; Age = 25; Job = "Teacher"; City = "Cannes"}
```

We used the "`;`" character between the different changes we imposed to jean. This is the *sequencing* of evaluations: it permits to evaluate successively several expressions, discarding the result of each (except the last one). This construct becomes useful in the presence of *side-effects* such as physical modifications and input/output, since we want to explicitly specify the order in which they are performed.

7.2 The ref type

The ref type is the predefined type of mutable indirection cells. It is present in the Caml system for reasons of compatibility with earlier versions of Caml. The ref type could be defined as follows (we don't use the ref name in the following definition because we want to preserve the original ref type):

```
#type 'a reference = {mutable Ref: 'a};;
Type reference defined.
```

Example of building a value of type `ref`:

```
#let r = ref (1+2);;
r : int ref = ref 3
```

The `ref` identifier is syntactically presented as a sum data constructor. The definition of `r` should be read as "let `r` be a reference to the value of `1+2`". The value of `r` is nothing but a memory location whose contents can be overwritten.

We consult a reference (i.e. read its memory location) with the "`!`" symbol:

```
#!r + 1;;
- : int = 4
```

We modify values of type `ref` with the `:=` infix function:

```
#r:=!r+1;;
- : unit = ()

#r;;
- : int ref = ref 4
```

Some primitives are attached to the `ref` type, for example:

```
#incr;;
- : int ref -> unit = <fun>

#decr;;
- : int ref -> unit = <fun>
```

which increments (resp. decrements) references on integers.

7.3 Arrays

Arrays are modifiable data structures. They belong to the parameterized type `'a vect`. Array expressions are bracketed by `[|` and `|]`, and elements are separated by semicolons:

```
#let a = [| 10; 20; 30 |];;
a : int vect = [|10; 20; 30|]
```

The length of an array is returned by with the function `vect_length`:

```
#vect_length a;;
- : int = 3
```

7.3.1 Accessing array elements

Accesses to array elements can be done using the following syntax:

```
#a.(0);;
- : int = 10
```

or, more generally: e_1`.(`e_2`)`, where e_1 evaluates to an array and e_2 to an integer. Alternatively, the function `vect_item` is provided:

```
#vect_item;;
- : 'a vect -> int -> 'a = <fun>
```

The first element of an array is at index 0. Arrays are useful because accessing an element is done in constant time: an array is a contiguous fragment of memory, while accessing list elements takes linear time.

7.3.2 Modifying array elements

Modification of an array element is done with the construct:

$$e_1\texttt{.(}e_2\texttt{) <- }e_3$$

where e_3 has the same type as the elements of the array e_1. The expression e_2 computes the index at which the modification will occur.

As for accessing, a function for modifying array elements is also provided:

```
#vect_assign;;
- : 'a vect -> int -> 'a -> unit = <fun>
```

For example:

```
#a.(0) <- (a.(0)-1);;
- : unit = ()

#a;;
- : int vect = [|9; 20; 30|]

#vect_assign a 0 ((vect_item a 0) - 1);;
- : unit = ()

#a;;
- : int vect = [|8; 20; 30|]
```

7.4 Loops: `while` and `for`

Imperative programming (i.e. using side-effects such as physical modification of data structures) traditionally makes use of sequences and explicit loops. Sequencing evaluation in Caml Light is done by using the semicolon "`;`". Evaluating expression e_1, discarding the value returned, and then evaluating e_2 is written:

$$e_1 \texttt{ ; } e_2$$

If e_1 and e_2 perform side-effects, this construct ensures that they will be performed in the specified order (from left to right). In order to emphasize sequential side-effects, instead of using parentheses around sequences, one can use `begin` and `end`, as in:

```
#let x = ref 1 in
#  begin
#     x := !x + 1;
#     x := !x * !x
#  end;;
- : unit = ()
```

The keywords `begin` and `end` are equivalent to opening and closing parentheses. The program above could be written as:

```
#let x = ref 1 in
#  (x := !x + 1; x := !x * !x);;
- : unit = ()
```

Explicit loops are not strictly necessary *per se*: a recursive function could perform the same work. However, the usage of an explicit loop locally emphasizes a more imperative style. Two loops are provided:

- *while*: `while` e_1 `do` e_2 `done` evaluates e_1 which must return a boolean expression, if e_1 return `true`, then e_2 (which is usually a sequence) is evaluated, then e_1 is evaluated again and so on until e_1 returns `false`.
- *for*: two variants, increasing and decreasing
  - `for` v`=`e_1 `to` e_2 `do` e_3 `done`
  - `for` v`=`e_1 `downto` e_2 `do` e_3 `done`

  where v is an identifier. Expressions e_1 and e_2 are the bounds of the loop: they must evaluate to integers. In the case of the increasing loop, the expressions e_1 and e_2 are evaluated producing values n_1 and n_2 : if n_1 is strictly greater than n_2, then nothing is done. Otherwise, e_3 is evaluated $(n_2 - n_1) + 1$ times, with the variable v bound successively to n_1, $n_1 + 1$, ..., n_2.

  The behavior of the decreasing loop is similar: if n_1 is strictly smaller than n_2, then nothing is done. Otherwise, e_3 is evaluated $(n_1 - n_2) + 1$ times with v bound to successive values decreasing from n_1 to n_2.

Both loops return the value `()` of type `unit`.

```
#for i=0 to (vect_length a) - 1 do a.(i) <- i done;;
- : unit = ()

#a;;
- : int vect = [|0; 1; 2|]
```

7.5 Polymorphism and mutable data structures

There are some restrictions concerning polymorphism and mutable data structures. One cannot enclose polymorphic objects inside mutable data structures.

```
#let r = ref [];;
r : '_a list ref = ref []
```

The reason is that once the type of `r`, `('a list) ref`, has been computed, it cannot be changed. But the value of `r` can be changed: we could write:

```
r:=[1;2];;
```

and now, `r` would be a reference on a list of numbers while its type would still be `('a list) ref`, allowing us to write:

```
r:= true::!r;;
```

making `r` a reference on `[true; 1; 2]`, which is an illegal Caml object.

Thus the Caml typechecker imposes that modifiable data structures appearing at toplevel must possess monomorphic types (i.e. not polymorphic).

Exercises

Exercise 7.1 *Give a mutable data type defining the Lisp type of lists and define the four functions* `car`, `cdr`, `rplaca` *and* `rplacd`.

Exercise 7.2 *Define a* `stamp` *function, of type* `unit -> int`, *that returns a fresh integer each time it is called. That is, the first call returns 1; the second call returns 2; and so on.*

Exercise 7.3 *Define a* `quick_sort` *function on arrays of floating point numbers following the* quicksort *algorithm [13]. Information about the* quicksort *algorithm can be found in [33], for example.*

Chapter 8

Escaping from computations: exceptions

In some situations, it is necessary to abort computations. If we are trying to compute the integer division of an integer `n` by `0`, then we must escape from that embarrassing situation without returning any result.

Another example of the usage of such an escape mechanism appears when we want to define the `head` function on lists:

```
#let head = function
#     x::L -> x
#   | [] -> raise (Failure "head: empty list");;
Toplevel input:
>     x::L -> x
>        ^
Warning: the variable L starts with an upper case letter in this pattern.
head : 'a list -> 'a = <fun>
```

We cannot give a regular value to the expression `head []` without losing the polymorphism of `head`. We thus choose to escape: we *raise an exception*.

8.1 Exceptions

An exception is a Caml value of the built-in type `exn`, very similar to a sum type. An exception:

- has a *name* (`Failure` in our example),
- and holds zero or one value (`"head:  empty list"` of type `string` in the example).

Some exceptions are predefined, like `Failure`. New exceptions can be defined with the following construct:

```
exception <exception name> [of <type>]
```

Example:

```
#exception Found of int;;
Exception Found defined.
```

The exception `Found` has been declared, and it carries integer values. When we apply it to an integer, we get an exception value (of type `exn`):

```
#Found 5;;
- : exn = Found 5
```

8.2 Raising an exception

Raising an exception is done by applying the primitive function `raise` to a value of type `exn`:

```
#raise;;
- : exn -> 'a = <fun>

#raise (Found 5);;
Uncaught exception: Found 5
```

Here is a more realistic example:

```
#let find_index p =
#  let rec find n =
#    function [] -> raise (Failure "not found")
#           | x::L -> if p(x) then raise (Found n)
#                       else find (n+1) L
#  in find 1;;
Toplevel input:
>          | x::L -> if p(x) then raise (Found n)
>                ^
Warning: the variable L starts with an upper case letter in this pattern.
find_index : ('a -> bool) -> 'a list -> 'b = <fun>
```

The `find_index` function always fails. It raises:

- `Found n`, if there is an element `x` of the list such that `p(x)`, in this case `n` is the index of `x` in the list,
- the `Failure` exception if no such `x` has been found.

Raising exceptions is more than an error mechanism: it is a programmable control structure. In the `find_index` example, there was no error when raising the `Found` exception: we only wanted to quickly escape from the computation, since we found what we were looking for. This is why it must be possible to *trap* exceptions: we want to trap possible errors, but we also want to get our result in the case of the `find_index function`.

8.3 Trapping exceptions

Trapping exceptions is achieved by the following construct:

```
try <expression> with <match cases>
```

This construct evaluates `<expression>`. If no exception is raised during the evaluation, then the result of the `try` construct is the result of `<expression>`. If an exception is raised during this evaluation, then the raised exception is matched against the `<match cases>`. If a case matches, then control is passed to it. If no case matches, then the exception is propagated outside of the `try` construct, looking for the enclosing `try`.

Example:

```
#let find_index p L =
#  let rec find n =
#    function [] -> raise (Failure "not found")
#           | x::L -> if p(x) then raise (Found n)
#                     else find (n+1) L
#  in
#    try find 1 L with Found n -> n;;
Toplevel input:
>let find_index p L =
>                 ^
Warning: the variable L starts with an upper case letter in this pattern.
Toplevel input:
>           | x::L -> if p(x) then raise (Found n)
>                ^
Warning: the variable L starts with an upper case letter in this pattern.
find_index : ('a -> bool) -> 'a list -> int = <fun>

#find_index (function n -> (n mod 2) = 0) [1;3;5;7;9;10];;
- : int = 6

#find_index (function n -> (n mod 2) = 0) [1;3;5;7;9];;
Uncaught exception: Failure "not found"
```

The `<match cases>` part of the `try` construct is a regular pattern matching on values of type `exn`. It is thus possible to trap any exception by using the `_` symbol. As an example, the following function traps any exception raised during the application of its two arguments. Warning: the `_` will also trap interrupts from the keyboard such as control-C!

```
#let catch_all f arg default =
#      try f(arg) with _ -> default;;
catch_all : ('a -> 'b) -> 'a -> 'b -> 'b = <fun>
```

It is even possible to catch all exceptions, do something special (close or remove opened files, for example), and raise again that exception, to propagate it upwards.

```
#let show_exceptions f arg =
#       try f(arg) with x -> print_string "Exception raised!\n"; raise x;;
show_exceptions : ('a -> 'b) -> 'a -> 'b = <fun>
```

In the example above, we print a message to the standard output channel (the terminal), before raising again the trapped exception.

```
#catch_all (function x -> raise (Failure "foo")) 1 0;;
- : int = 0
```

```
#catch_all (show_exceptions (function x -> raise (Failure "foo"))) 1 0;;
Exception raised!
- : int = 0
```

8.4 Polymorphism and exceptions

Exceptions must not be polymorphic for a reason similar to the one for references (although it is a bit harder to give an example).

```
#exception Exc of 'a list;;
Toplevel input:
>exception Exc of 'a list;;
>                 ^^
The type variable a is unbound.
```

One reason is that the `excn` type is not a parameterized type, but one deeper reason is that if the exception `Exc` is declared to be polymorphic, then a function may raise `Exc [1;2]`. There might be no mention of that fact in the type inferred for the function. Then, another function may trap that exception, obtaining the value `[1;2]` whose real type is `int list`. But the only type known by the typechecker is `'a list`: the `try` form should refer to the `Exc` data constructor, which is known to be polymorphic. It may then be possible to build an ill-typed Caml value `[true; 1; 2]`, since the typechecker does not possess any further type information than `'a list`.

The problem is thus the absence of static connection from exceptions that are raised and the occurrences where they are trapped. Another example would be the one of a function raising `Exc` with an integer or a boolean value, depending on some condition. Then, in that case, when trying to trap these exceptions, we cannot decide wether they will hold integers or boolean values.

Exercises

Exercise 8.1 *Define the function* `find_succeed` *which given a function* `f` *and a list* `L` *returns the first element of* `L` *on which the application of* `f` *succeeds.*

Exercise 8.2 *Define the function* `map_succeed` *which given a function* `f` *and a list* `L` *returns the list of the results of successful applications of* `f` *to elements of* `L`*.*

Chapter 9

Basic input/output

We describe in this chapter the Caml Light input/output model and some of its primitive operations. More complete information about IO can be found in the Caml Light manual [21].

Caml Light has an imperative input/output model: an IO operation should be considered as a side-effect, and is thus dependent on the order of evaluation. IOs are performed onto *channels* with types `in_channel` and `out_channel`. These types are *abstract*, i.e. their representation is not accessible.

Three channels are predefined:

```
#std_in;;
- : in_channel = <abstr>

#std_out;;
- : out_channel = <abstr>

#std_err;;
- : out_channel = <abstr>
```

They are the "standard" IO channels: `std_in` is usually connected to the keyboard, and printing onto `std_out` and `std_err` usually appears on the screen.

9.1 Printable types

It is not possible to print and read every value. Functions, for example, are typically not readable, unless a suitable string representation is designed and reading such a representation is followed by an interpretation computing the desired function.

We call *printable type* a type for which there are input/output primitives implemented in Caml Light. The main printable types are:

- characters: type `char`;
- strings: type `string`;
- integers: type `int`;
- floating point numbers: type `float`.

We know all these types from the previous chapters. Strings and characters support a notation for escaping to ASCII codes or to denote special characters such as newline:

```
#`A`;;
- : char = `A`

#`\065`;;
- : char = `A`

#`\\`;;
- : char = `\\`

#`\n`;;
- : char = `\n`

#"string with\na newline inside";;
- : string = "string with\na newline inside"
```

The "\" character is used as an escape and is useful for non-printable or special characters.

Of course, character constants can be used as constant patterns:

```
#function `a` -> 0 | _ -> 1;;
- : char -> int = <fun>
```

On types such as `char` that have a finite number of constant elements, it may be useful to use *or-patterns*, gathering constants in the same matching rule:

```
#let is_vowel = function
#  `a` | `e` | `i` | `o` | `u` | `y` -> true
#| _ -> false;;
is_vowel : char -> bool = <fun>
```

The first rule is chosen if the argument matches one of the cases. Since there is a total ordering on characters, the syntax of character patterns is enriched with a "`..`" notation:

```
#let is_lower_case_letter = function
#  `a`..`z` -> true
#| _ -> false;;
is_lower_case_letter : char -> bool = <fun>
```

Of course, or-patterns and this notation can be mixed, as in:

```
#let is_letter = function
#  `a`..`z` | `A`..`Z` -> true
#| _ -> false;;
is_letter : char -> bool = <fun>
```

In the next sections, we give the most commonly used IO primitives on these printable types. A complete listing of predefined IO operations is given in [21].

9.2 Output

Printing on standard output is performed by the following functions:

```
#print_char;;
- : char -> unit = <fun>
```

```
#print_string;;
- : string -> unit = <fun>
```

```
#print_int;;
- : int -> unit = <fun>
```

```
#print_float;;
- : float -> unit = <fun>
```

Printing is *buffered*, i.e. the effect of a call to a printing function may not be seen immediately: *flushing* explicitly the output buffer is sometimes required, unless a printing function flushes it implicitly. Flushing is done with the `flush` function:

```
#flush;;
- : out_channel -> unit = <fun>
```

```
#print_string "Hello!"; flush std_out;;
Hello!- : unit = ()
```

The `print_newline` function prints a newline character and flushes the standard output:

```
#print_newline;;
- : unit -> unit = <fun>
```

Flushing is required when writing standalone applications, in which the application may terminate without all printing being done. Standalone applications should terminate by a call to the `exit` function (from the `io` module), which flushes all pending output on `std_out` and `std_err`.

Printing on the standard error channel `std_err` is done with the following functions:

```
#prerr_char;;
- : char -> unit = <fun>
```

```
#prerr_string;;
- : string -> unit = <fun>
```

```
#prerr_int;;
- : int -> unit = <fun>
```

```
#prerr_float;;
- : float -> unit = <fun>
```

The following function prints its string argument followed by a newline character to `std_err` and then flushes `std_err`.

```
#prerr_endline;;
- : string -> unit = <fun>
```

9.3 Input

These input primitives flush the standard output and read from the standard input:

```
#read_line;;
- : unit -> string = <fun>

#read_int;;
- : unit -> int = <fun>

#read_float;;
- : unit -> float = <fun>
```

Because of their names and types, these functions do not need further explanation.

9.4 Channels on files

When programs have to read from or print to files, it is necessary to open and close channels on these files.

9.4.1 Opening and closing channels

Opening and closing is performed with the following functions:

```
#open_in;;
- : string -> in_channel = <fun>

#open_out;;
- : string -> out_channel = <fun>

#close_in;;
- : in_channel -> unit = <fun>

#close_out;;
- : out_channel -> unit = <fun>
```

The `open_in` function checks the existence of its filename argument, and returns a new input channel on that file; `open_out` creates a new file (or truncates it to zero length if it exists) and returns an output channel on that file. Both functions fail if permissions are not sufficient for reading or writing.
Warning:

- Closing functions close their channel argument. Since their behavior is unspecified on already closed channels, anything can happen in this case!

- Closing one of the standard IO channels (`std_in`, `std_out`, `std_err`) have unpredictable effects!

9.4.2 Reading or writing from/to specified channels

Some of the functions on standard input/output have corresponding functions working on channels:

```
#output_char;;
- : out_channel -> char -> unit = <fun>
```

```
#output_string;;
- : out_channel -> string -> unit = <fun>
```

```
#input_char;;
- : in_channel -> char = <fun>
```

```
#input_line;;
- : in_channel -> string = <fun>
```

9.4.3 Failures

The exception `End_of_file` is raised when an input operation cannot complete because the end of the file has been reached.

```
#End_of_file;;
- : exn = End_of_file
```

The exception `sys__Sys_error` (`Sys_error` from the module `sys`) is raised when some manipulation of files is forbidden by the operating system:

```
#open_in "abracadabra";;
Uncaught exception: sys__Sys_error "abracadabra: No such file or directory"
```

The functions that we have seen in this chapter are sufficient for our needs. Many more exist (useful mainly when working with files) and are described in [21].

Exercises

Exercise 9.1 *Define a function* `copy_file` *taking two filenames (of type* `string`*) as arguments, and copying the contents of the first file on the second one. Error messages must be printed on* `std_err`*.*

Exercise 9.2 *Define a function* `wc` *taking a filename as argument and printing on the standard output the number of characters and lines appearing in the file. Error messages must be printed on* `std_err`*.*

Note: it is good practice to develop a program in defining small functions. A single function doing the whole work is usually harder to debug and to read. With small functions, one can trace them and see the arguments they are called on and the result they produce.

Chapter 10

Streams and parsers

In the next part of these course notes, we will implement a small functional language. Parsing valid programs of this language requires writing a lexical analyzer and a parser for the language. For the purpose of writing easily such programs, Caml Light provides a special data structure: *streams*. Their main usage is to be interfaced to input channels or strings and to be matched against *stream patterns*.

10.1 Streams

Streams belong to an abstract data type: their actual representation remains hidden from the user. However, it is still possible to build streams either "by hand" or by using some predefined functions.

10.1.1 The `stream` type

The type `stream` is a parameterized type. One can build streams of integers, of characters or of any other type. Streams receive a special syntax, looking like the one for lists. The empty stream is written:

```
#[< >];;
- : '_a stream = <abstr>
```

A non empty stream possesses elements that are written preceded by the "'" (quote) character.

```
#[< '0; '1; '2 >];;
- : int stream = <abstr>
```

Elements that are not preceded by "'" are *substreams* that are expanded in the enclosing stream:

```
#[< '0; [<'1;'2>]; '3 >];;
- : int stream = <abstr>

#let s = [< '"abc" >] in [< s; '"def" >];;
- : string stream = <abstr>
```

Thus, stream concatenation can be defined as:

```
#let stream_concat s t = [< s; t >];;
stream_concat : 'a stream -> 'a stream -> 'a stream = <fun>
```

Building streams in this way can be useful while testing a parsing function or defining a lexical analyzer (taking as argument a stream of characters and returning a stream of tokens). Stream concatenation *does not copy* substreams: they are simply put in the same stream. Since (as we will see later) stream matching has a destructive effect on streams (streams are physically "eaten" by stream matching), parsing `[< t; t >]` will in fact parse `t` only once: the first occurrence of `t` will be consumed, and the second occurrence will be empty before its parsing will be performed.

Interfacing streams with an input channel can be done with the function:

```
#stream_of_channel;;
- : in_channel -> char stream = <fun>
```

returning a stream of characters which are read from the channel argument. The end of stream will coincide with the end of the file associated to the channel.

In the same way, one can build the character stream associated to a character string using:

```
#stream_of_string;;
- : string -> char stream = <fun>

#let s = stream_of_string "abc";;
s : char stream = <abstr>
```

10.1.2 Streams are lazily evaluated

Stream expressions are submitted to *lazy evaluation*, i.e. they are effectively build only when required. This is useful in that it allows for the easy manipulation of "interactive" streams like the stream built from the standard input. If this was not the case, i.e. if streams were immediately completely computed, a program evaluating "`stream_of_channel std_in`" would read everything up to an end-of-file on standard input before giving control to the rest of the program. Furthermore, lazy evaluation of streams allows for the manipulation of infinite streams. As an example, we can build the infinite stream of integers, using side effects to show precisely when computations occur:

```
#let rec ints_from n =
#   [< '(print_int n; print_char ` `; flush std_out; n);
#      ints_from (n+1) >];;
ints_from : int -> int stream = <fun>

#let ints = ints_from 0;;
ints : int stream = <abstr>
```

We notice that no printing occurred and that the program terminates: this shows that none of the elements have been evaluated and that the infinite stream has not been built. We will see in the next section that these side-effects will occur on demand, i.e. when tests will be needed by a matching function on streams.

10.2 Stream matching and parsers

The syntax for building streams can be used for pattern-matching over them. However, stream matching is more complex than the usual pattern matching.

10.2.1 Stream matching is destructive

Let us start with a simple example:

```
#let next = function [< 'x >] -> x;;
next : 'a stream -> 'a = <fun>
```

The `next` function returns the first element of its stream argument, and fails if the stream is empty:

```
#let s = [< '0; '1; '2 >];;
s : int stream = <abstr>

#next s;;
- : int = 0

#next s;;
- : int = 1

#next s;;
- : int = 2

#next s;;
Uncaught exception: Parse_failure
```

We can see from the previous examples that the stream pattern `[< 'x >]` matches *an initial segment* of the stream. Such a pattern must be read as "the stream whose first element matches `x`". Furthermore, once stream matching has succeeded, the stream argument has been *physically modified* and does not contain any longer the part that has been recognized by the `next` function.

If we come back to the infinite stream of integers, we can see that the calls to `next` provoke the evaluation of the necessary part of the stream:

```
#next ints; next ints; next ints;;
Toplevel input:
>next ints; next ints; next ints;;
>^^^^^^^^^
Warning: this expression has type int,
but is used with type unit.
Toplevel input:
>next ints; next ints; next ints;;
>           ^^^^^^^^^
Warning: this expression has type int,
but is used with type unit.
0 1 2 - : int = 2
```

Thus, successive calls to `next` remove the first elements of the stream until it becomes empty. Then, `next` fails when applied to the empty stream, since, in the definition of `next`, there is no stream pattern that matches an initial segment of the empty stream.

It is of course possible to specify several stream patterns as in:

```
#let next = function
#  [< 'x >] -> x
#| [< >] -> raise (Failure "empty");;
next : 'a stream -> 'a = <fun>
```

Cases are tried in turn, from top to bottom.

Stream pattern components are not restricted to quoted patterns (intended to match stream elements), but can be also function calls (corresponding to non-terminals, in the grammar terminology). Functions appearing as stream pattern components are intended to match substreams of the stream argument: they are called on the actual stream argument, and they are followed by a pattern which should match the result of this call. For example, if we define a parser recognizing a non empty sequence of characters `'a'`:

```
#let seq_a =
#    let rec seq = function
#        [< ''a'; seq l >] -> 'a'::l
#      | [< >] -> []
#    in function [< ''a'; seq l >] -> 'a'::l;;
seq_a : char stream -> char list = <fun>
```

we used the recursively defined function `seq` inside the stream pattern of the first rule. This definition should be read as:

- if the stream is not empty and if its first element matches `'a'`, apply `seq` to the rest of the stream, let `l` be the result of this call and return `'a'::l`,
- otherwise, fail (raise `Parse_failure`);

and `seq` should be read in the same way (except that, since it recognizes possibly empty sequences of `'a'`, it never fails).

Less operationally, we can read it as: "a non-empty sequence of `'a'` starts with an `'a'`, and is followed by a possibly empty sequence of `'a'`.

Another example is the recognition of a non-empty sequence of `'a'` followed by a `'b'`, or a `'b'` alone:

```
#let seq_a_b = function
#  [< seq_a l; ''b' >] -> l@['b']
#| [< ''b' >] -> ['b'];;
seq_a_b : char stream -> char list = <fun>
```

Here, operationally, once an `'a'` has been recognized, the first matching rule is chosen. Any further mismatch (either from `seq_a` or from the last `'b'`) will raise a `Parse_error` exception, and the whole parsing will fail. On the other hand, if the first character is not an `'a'`, `seq_a` will raise `Parse_failure`, and the second rule (`[< ''b' >] -> ...`) will be tried.

This behavior is typical of predictive parsers. Predictive parsing is recursive-descent parsing with one look-ahead token. In other words, a predictive parser is a set of (possibly mutually recursive) procedures, which are selected according to the shape of (at most) the first token.

10.2.2 Sequential binding in stream patterns

Bindings in stream patterns occur sequentially, in contrast with bindings in regular patterns, which can be thought as occurring in parallel. Stream matching is guaranteed to be performed from left to right. For example, computing the sum of the elements of an integer stream could be defined as:

```
#let rec stream_sum n = function
#  [< '0; (stream_sum n) p >] -> p
#| [< 'x; (stream_sum (n+x)) p >] -> p
#| [< >] -> n;;
stream_sum : int -> int stream -> int = <fun>

#stream_sum 0 [< '0; '1; '2; '3; '4 >];;
- : int = 10
```

The `stream_sum` function uses its first argument as an accumulator holding the sum computed so far. The call `(stream_sum (n+x))` uses `x` which was bound in the stream pattern component occurring at the left of the call.

Warning: streams patterns are legal only in the `function` and `match` constructs. The `let` and other forms are restricted to usual patterns. Furthermore, a stream pattern cannot appear inside another pattern.

10.3 Parameterized parsers

Since a parser is a function like any other function, it can be parameterized or be used as a parameter. Parameters used only in the right-hand side of stream-matching rules simulate *inherited attributes* of attribute grammars. Parameters used as parsers in stream patterns allow for the implementation of *higher-order* parsers. We will use the next example to motivate the introduction of parameterized parsers.

10.3.1 Example: a parser for arithmetic expressions

Before building a parser for arithmetic expressions, we need a lexical analyzer able to recognize arithmetic operations and integer constants. Let us first define a type for tokens:

```
#type token =
#  PLUS | MINUS | TIMES | DIV | LPAR | RPAR
#| INT of int;;
Type token defined.
```

Skipping blank spaces is performed by the `spaces` function defined as:

```
#let rec spaces = function
#  [< '` `|`\t`|`\n`; spaces _ >] -> ()
```

```
#| [< >] -> ();;
spaces : char stream -> unit = <fun>
```

The conversion of a digit (character) into its integer value is done by:

```
#let int_of_digit = function
#  `0`..`9` as c -> (int_of_char c) - (int_of_char `0`)
#| _ -> raise (Failure "not a digit");;
int_of_digit : char -> int = <fun>
```

The "as" keyword allows for naming a pattern: in this case, the variable c will be bound to the actual digit matched by `0`..`9`. Pattern built with as are called *alias patterns*.

For the recognition of integers, we already feel the need for a parameterized parser. Integer recognition is done by the integer analyzer defined below. It is parameterized by a numeric value representing the value of the first digits of the number:

```
#let rec integer n = function
#  [< ' `0`..`9` as c; (integer (10*n + int_of_digit c)) r >] -> r
#| [< >] -> n;;
integer : int -> char stream -> int = <fun>

#integer 0 (stream_of_string "12345");;
- : int = 12345
```

We are now ready to write the lexical analyzer, taking a stream of characters, and returning a stream of tokens. Returning a token stream which will be explored by the parser is a simple, reasonably efficient and intuitive way of composing a lexical analyzer and a parser.

```
#let rec lexer s = match s with
#  [< '`(`; spaces _ >] -> [< 'LPAR; lexer s >]
#| [< '`)`; spaces _ >] -> [< 'RPAR; lexer s >]
#| [< '`+`; spaces _ >] -> [< 'PLUS; lexer s >]
#| [< '`-`; spaces _ >] -> [< 'MINUS; lexer s >]
#| [< '`*`; spaces _ >] -> [< 'TIMES; lexer s >]
#| [< '`/`; spaces _ >] -> [< 'DIV; lexer s >]
#| [< '`0`..`9` as c; (integer (int_of_digit c)) n; spaces _ >]
#                        -> [< 'INT n; lexer s >];;
lexer : char stream -> token stream = <fun>
```

We assume there is no leading space in the input.

Now, let us examine the language that we want to recognize. We shall have integers, infix arithmetic operations and parenthesized expressions. The BNF form of the grammar is:

```
Expr ::= Expr + Expr
       | Expr - Expr
       | Expr * Expr
       | Expr / Expr
       | ( Expr )
       | INT
```

The values computed by the parser will be *abstract syntax trees* (by contrast with *concrete syntax*, which is the input string or stream). Such trees belong to the following type:

```
#type atree =
#  Int of int
#| Plus of atree * atree
#| Minus of atree * atree
#| Mult of atree * atree
#| Div of atree * atree;;
Type atree defined.
```

The `Expr` grammar is ambiguous. To make it unambiguous, we will adopt the usual precedences for arithmetic operators and assume that all operators associate to the left. Now, to use stream matching for parsing, we must take into account the fact that matching rules are chosen according to the behavior of the first component of each matching rule. This is predictive parsing, and, using well-known techniques, it is easy to rewrite the grammar above in such a way that writing the corresponding predictive parser becomes trivial. These techniques are described in [2], and consist in adding a non-terminal for each precedence level, and removing left-recursion. We obtain:

```
Expr ::= Mult
       | Mult + Expr
       | Mult - Expr

Mult ::= Atom
       | Atom * Mult
       | Atom / Mult

Atom ::= INT
       | ( Expr )
```

While removing left-recursion, we forgot about left associativity of operators. This is not a problem, as long as we build correct abstract trees.

Since stream matching bases its choices on the first component of stream patterns, we cannot see the grammar above as a parser. We need a further transformation, factoring common prefixes of grammar rules (left-factor). We obtain:

```
Expr ::= Mult RestExpr

        RestExpr ::= + Mult RestExpr
                   | - Mult RestExpr
                   | (* nothing *)

Mult ::= Atom RestMult

        RestMult ::= * Atom RestMult
                   | / Atom RestMult
                   | (* nothing *)
```

```
Atom ::= INT
       | ( Expr )
```

Now, we can see this grammar as a parser (note that the order of rules becomes important, and empty productions must appear last). The shape of the parser is:

```
let rec expr =
    let rec restexpr ? = function
        [< 'PLUS; mult ?; restexpr ? >] -> ?
      | [< 'MINUS; mult ?; restexpr ? >] -> ?
      | [< >] -> ?
in function [< mult e1; restexpr ? >] -> ?

and mult =
    let rec restmult ? = function
        [< 'TIMES; atom ?; restmult ? >] -> ?
      | [< 'DIV; atom ?; restmult ? >] -> ?
      | [< >] -> ?
in function [< atom e1; restmult ? >] -> ?

and atom = function
  [< 'INT n >] -> Int n
| [< 'LPAR; expr e; 'RPAR >] -> e
```

We used question marks where parameters, bindings and results still have to appear. Let us consider the `expr` function: clearly, as soon as `e1` is recognized, we must be ready to build the leftmost subtree of the result. This leftmost subtree is either restricted to `e1` itself, in case `restexpr` does not encounter any operator, or it is the tree representing the addition (or subtraction) of `e1` and the expression immediately following the additive operator. Therefore, `restexpr` must be called with `e1` as an intermediate result, and accumulate subtrees built from its intermediate result, the tree constructor corresponding to the operator and the last expression encountered. The body of `expr` becomes:

```
let rec expr =
    let rec restexpr e1 = function
        [< 'PLUS; mult e2; restexpr (Plus (e1,e2)) e >] -> e
      | [< 'MINUS; mult e2; restexpr (Minus (e1,e2)) e >] -> e
      | [< >] -> e1
in function [< mult e1; (restexpr e1) e2 >] -> e2
```

Now, `expr` recognizes a product `e1` (by `mult`), and applies `(restexpr e1)` to the rest of the stream. According to the additive operator encountered (if any), this function will apply `mult` which will return some `e2`. Then the process continues with `Plus(e1,e2)` as intermediate result. In the end, a correctly balanced tree will be produced (using the last rule of `restexpr`).

With the same considerations on `mult` and `restmult`, we can complete the parser, obtaining:

```
#let rec expr =
#    let rec restexpr e1 = function
#        [< 'PLUS; mult e2; (restexpr (Plus (e1,e2))) e >] -> e
#      | [< 'MINUS; mult e2; (restexpr (Minus (e1,e2))) e >] -> e
#      | [< >] -> e1
#in function [< mult e1; (restexpr e1) e2 >] -> e2
#
#and mult =
#    let rec restmult e1 = function
#        [< 'TIMES; atom e2; (restmult (Mult (e1,e2))) e >] -> e
#      | [< 'DIV; atom e2; (restmult (Div (e1,e2))) e >] -> e
#      | [< >] -> e1
#in function [< atom e1; (restmult e1) e2 >] -> e2
#
#and atom = function
#  [< 'INT n >] -> Int n
#| [< 'LPAR; expr e; 'RPAR >] -> e;;
expr : token stream -> atree = <fun>
mult : token stream -> atree = <fun>
atom : token stream -> atree = <fun>
```

And we can now try our parser:

```
#expr (lexer (stream_of_string "(1+2+3*4)-567"));;
- : atree = Minus (Plus (Plus (Int 1, Int 2), Mult (Int 3, Int 4)), Int 567)
```

10.3.2 Parameters simulating inherited attributes

In the previous example, the parsers `restexpr` and `restmult` take an abstract syntax tree `e1` as argument and pass it down to the result through recursive calls such as `(restexpr (Plus(e1,e2)))`. If we see such parsers as non-terminals (`RestExpr` from the grammar above) this parameter acts as an inherited attribute of the non-terminal. Synthesized attributes are simulated by the right hand sides of stream matching rules.

10.3.3 Higher-order parsers

In the definition of `expr`, we may notice that the parsers `expr` and `mult` on the one hand and `restexpr` and `restmult` on the other hand have exactly the same structure. To emphasize this similarity, if we define parsers for additive (resp. multiplicative) operators by:

```
#let addop = function
#  [< 'PLUS >] -> (function (x,y) -> Plus(x,y))
#| [< 'MINUS >] -> (function (x,y) -> Minus(x,y))
#and multop = function
#  [< 'TIMES >] -> (function (x,y) -> Mult(x,y))
#| [< 'DIV >] -> (function (x,y) -> Div(x,y));;
```

```
addop : token stream -> atree * atree -> atree = <fun>
multop : token stream -> atree * atree -> atree = <fun>
```

we can rewrite the `expr` parser as:

```
#let rec expr =
#    let rec restexpr e1 = function
#        [< addop f; mult e2; (restexpr (f (e1,e2))) e >] -> e
#      | [< >] -> e1
#in function [< mult e1; (restexpr e1) e2 >] -> e2
#
#and mult =
#    let rec restmult e1 = function
#        [< multop f; atom e2; (restmult (f (e1,e2))) e >] -> e
#      | [< >] -> e1
#in function [< atom e1; (restmult e1) e2 >] -> e2
#
#and atom = function
#  [< 'INT n >] -> Int n
#| [< 'LPAR; expr e; 'RPAR >] -> e;;
expr : token stream -> atree = <fun>
mult : token stream -> atree = <fun>
atom : token stream -> atree = <fun>
```

Now, we take advantage of these similarities in order to define a general parser for left-associative operators. Its name is `left_assoc` and is parameterized by a parser for operators and a parser for expressions:

```
#let rec left_assoc op term =
#    let rec rest e1 = function
#        [< op f; term e2; (rest (f (e1,e2))) e >] -> e
#      | [< >] -> e1
#    in function [< term e1; (rest e1) e2 >] -> e2;;
left_assoc :
 ('a stream -> 'b * 'b -> 'b) -> ('a stream -> 'b) -> 'a stream -> 'b = <fun>
```

Now, we can redefine `expr` as:

```
#let rec expr str = left_assoc addop mult str
#and mult str = left_assoc multop atom str
#and atom = function
#  [< 'INT n >] -> Int n
#| [< 'LPAR; expr e; 'RPAR >] -> e;;
expr : token stream -> atree = <fun>
mult : token stream -> atree = <fun>
atom : token stream -> atree = <fun>
```

And we can now try our definitive parser:

```
#expr (lexer (stream_of_string "(1+2+3*4)-567"));;
- : atree = Minus (Plus (Plus (Int 1, Int 2), Mult (Int 3, Int 4)), Int 567)
```

Parameterized parsers are useful for defining general parsers such as `left_assoc` that can be used with different instances. Another example of a useful general parser is the `star` parser defined as:

```
#let rec star p = function
#  [< p x; (star p) l >] -> x::l
#| [< >] -> [];;
star : ('a stream -> 'b) -> 'a stream -> 'b list = <fun>
```

The `star` parser iterates zero or more times its argument `p` and returns the list of results. We still have to be careful in using these general parsers because of the predictive nature of parsing. For example, `star p` will never successfully terminate if `p` has a rule for the empty stream pattern: this rule will make the second rule of `star` useless!

10.3.4 Example: parsing a non context-free language

As an example of parsing with parameterized parsers, we shall build a parser for the language $\{wCw \mid w \in (A|B)^*\}$, which is known to be non context-free.

First, let us define a type for this alphabet:

```
#type token = A | B | C;;
Type token defined.
```

Given an input of the form wCw, the idea for a parser recognizing it is:

- first, to recognize the sequence w with a parser `wd` (for *word definition*) returning information in order to build a parser recognizing only w;
- then to recognize `C`;
- and to use the parser built at the first step to recognize the sequence w.

The definition of `wd` is as follows:

```
#let rec wd = function
#  [< 'A; wd l >] -> (function [< 'A >] -> "a")::l
#| [< 'B; wd l >] -> (function [< 'B >] -> "b")::l
#| [< >] -> [];;
wd : token stream -> (token stream -> string) list = <fun>
```

The `wu` function (for *word usage*) builds a parser sequencing a list of parsers:

```
#let rec wu = function
#  p::pl -> (function [< p x; (wu pl) l >] -> x^l)
#| [] -> (function [< >] -> "");;
wu : ('a stream -> string) list -> 'a stream -> string = <fun>
```

The `wu` function builds, from a list of parsers p_i, for $i = 1..n$, a single parser

$$\texttt{function [<}p_1\ x_1\texttt{;...;}p_n\ x_n\texttt{>] -> [}x_1\texttt{;...;}x_n\texttt{]}$$

which is the sequencing of parsers p_i. The main parser `w` is:

```
#let w = function [< wd l; 'C; (wu l) r >] -> r;;
w : token stream -> string = <fun>
#w [< 'A; 'B; 'B; 'C; 'A; 'B; 'B >];;
- : string = "abb"
#w [< 'C >];;
- : string = ""
```

In the previous parser, we used an intermediate list of parsers in order to build the second parser. We can redefine `wd` without using such a list:

```
#let w =
#    let rec wd wr = function
#        [< 'A; (wd (function [< wr r; 'A >] -> r^"a")) p >] -> p
#      | [< 'B; (wd (function [< wr r; 'B >] -> r^"b")) p >] -> p
#      | [< >] -> wr
#    in function [< (wd (function [< >] -> "")) p; 'C; p str >] -> str;;
w : token stream -> string = <fun>
#w [< 'A; 'B; 'B; 'C; 'A; 'B; 'B >];;
- : string = "abb"
#w [< 'C >];;
- : string = ""
```

Here, `wd` is made local to `w`, and takes as parameter `wr` (for *word recognizer*) whose initial value is the parser with an empty stream pattern. This parameter accumulates intermediate results, and is delivered at the end of parsing the initial sequence w. After checking for the presence of `C`, it is used to parse the second sequence w.

10.4 Further reading

A summary of the constructs over streams and of primitives over streams is given in [21].

An alternative to parsing with streams and stream matching are the `camllex` and `camlyacc` programs.

A detailed presentation of streams and stream matching following "predictive parsing" semantics can be found in [24], where alternative semantics are given with some possible implementations.

Exercises

Exercise 10.1 *Define a parser for the language of prefix arithmetic expressions generated by the grammar:*

```
Expr ::= INT
       | + Expr Expr
       | - Expr Expr
       | * Expr Expr
       | / Expr Expr
```

Use the lexical analyzer for arithmetic expressions given above. The result of the parser must be the integer resulting from the evaluation of the arithmetic expression, i.e. its type must be:

```
token -> int
```

Exercise 10.2 *Enrich the type* `token` *above with a constructor* `IDENT of string` *for identifiers, and enrich the lexical analyzer for it to recognize identifiers built from alphabetic letters (upper or lowercase). Length of identifiers may be limited.*

Chapter 11

Standalone programs and separate compilation

So far, we have used Caml Light in an interactive way. It is also possible to program in Caml Light in a batch-oriented way: writing source code in a file, having it compiled into an executable program, and executing the program outside of the Caml Light environment. Interactive use is great for learning the language and quickly testing new functions. Batch use is more convenient to develop larger programs, that should be usable without knowledge of Caml Light.

Note for Macintosh users: batch compilation is not available in the standalone Caml Light application. It requires the MPW environment (see the Caml Light manual).

11.1 Standalone programs

Standalone programs are composed of a sequence of phrases, contained in one or several text files. Phrases are the same as at toplevel: expressions, value declarations, type declarations, exception declarations, and directives. When executing the stand-alone program produced by the compiler, all phrases are executed in order. The values of expressions and declared global variables are not printed, however. A stand-alone program has to perform input and output explicitly.

Here is a sample program, that prints the number of characters and the number of lines of its standard input, like the `wc` Unix utility.

```
let chars = ref 0;;
let lines = ref 0;;
try
  while true do
    let c = input_char std_in in
      chars := !chars + 1;
      if c = `\n` then lines := !lines + 1 else ()
  done
with End_of_file ->
  print_int !chars; print_string " characters, ";
  print_int !lines; print_string " lines.\n";
  exit 0
```

```
;;
```

The `input_char` function reads the next character from an input channel (here, `std_in`, the channel connected to standard input). It raises exception `End_of_file` when reaching the end of the file. The `exit` function aborts the process. Its argument is the exit status of the process. Calling `exit` is absolutely necessary to ensure proper flushing of the output channels.

Assume this program is in file `count.ml`. To compile it, simply run the `camlc` command from the command interpreter:

```
camlc -o count count.ml
```

The compiler produces an executable file `count`. You can now run `count` with the help of the "camlrun" command:

```
camlrun count < count.ml
```

This should display something like:

```
306 characters, 13 lines.
```

Under Unix, the `count` file can actually be executed directly, just like any other Unix command, as in:

```
./count < count.ml
```

This also works under MS-DOS, provided the executable file is given extension `.exe`. That is, if we compile `count.ml` as follows:

```
camlc -o count.exe count.ml
```

we can run `count.exe` directly, as in:

```
count.exe < count.ml
```

See the reference manual for more information on `camlc`.

11.2 Programs in several files

It is possible to split one program into several source files, separately compiled. This way, local changes do not imply a full recompilation of the program. Let us illustrate that on the previous example. We split it in two modules: one that implements integer counters; another that performs the actual counting. Here is the first one, `counter.ml`:

```
(* counter.ml *)
type counter = { mutable val: int };;
let new init = { val = init };;
let incr c = c.val <- c.val + 1;;
let read c = c.val;;
```

Here is the source for the main program, in file `main.ml`.

```
(* main.ml *)
let chars = counter__new 0;;
let lines = counter__new 0;;
try
  while true do
    let c = input_char std_in in
      counter__incr chars;
      if c = `\n` then counter__incr lines else ()
  done
with End_of_file ->
  print_int (counter__read chars); print_string " characters, ";
  print_int (counter__read lines); print_string " lines.\n";
  exit 0
;;
```

Notice that references to identifiers defined in module `counter.ml` are prefixed with the name of the module, `counter`, and by `__` (the "long dash" symbol: two underscore characters). If we had simply entered `new 0`, for instance, the compiler would have assumed `new` is an identifier declared in the current module, and issued an "undefined identifier" error.

Compiling this program requires two compilation steps, plus one final linking step.

```
camlc -c counter.ml
camlc -c main.ml
camlc -o main counter.zo main.zo
```

Running the program is done as before:

```
camlrun main < counter.ml
```

The `-c` option to `camlc` means "compile only"; that is, the compiler should not attempt to produce a stand-alone executable program from the given file, but simply an object code file (files `counter.zo`, `main.zo`). The final linking phases takes the two `.zo` files and produces the executable `main`. Object files must be linked in the right order: for each global identifier, the module defining it must come before the modules that use it.

Prefixing all external identifiers by the name of their defining module is sometimes tedious. Therefore, the Caml Light compiler provides a mechanism to omit the `module__` part in external identifiers. The system maintains a list of "default" modules, called the currently opened modules, and whenever it encounters an identifier without the `module__` part, it searches through the opened modules, to find one that defines this identifier. Searched modules always include the module being compiled (searched first), and some library modules of general use. In addition, two directives are provided to add and to remove modules from the list of opened modules:

- `#open "module";;` to add `module` in front of the list;
- `#close "module";;` to remove `module` from the list.

For instance, we can rewrite the `main.ml` file above as:

```
#open "counter";;
let chars = new 0;;
let lines = new 0;;
try
  while true do
    let c = input_char std_in in
      incr chars;
      if c = `\n` then incr lines
  done
with End_of_file ->
  print_int (read chars);
  print_string " characters, ";
  print_int (read lines);
  print_string " lines.\n";
  exit 0
;;
```

After the `#open "counter"` directive, the identifier `new` automatically resolves to `counters__new`.

If two modules, say `mod1` and `mod2`, define both a global value `f`, then `f` in a client module `client` resolves to `mod1__f` if `mod1` is opened but not `mod2`, or if `mod1` has been opened more recently than `mod2`. Otherwise, it resolves to `mod2__f`. For instance, the two system modules `int` and `float` both define the infix identifier `+`. Both modules `int` and `float` are opened by default, but `int` comes first. Hence, `x + y` is understood as the integer addition, since `+` is resolved to `int__+`. But after the directive `#open "float";;`, module `float` comes first, and the identifier `+` is resolved to `float__+`.

11.3 Abstraction

Some globals defined in a module are not intended to be used outside of this module. Then, it is good programming style not to export them outside of the module, so that the compiler can check they are not used in another module. Also, one may wish to export a data type abstractly, that is, without publicizing the structure of the type. This ensures that other modules cannot build or inspect objects of that type without going through one of the functions on that type exported in the defining module. This helps in writing clean, well-structured programs.

The way to do that in Caml Light is to write an explicit interface, or output signature, specifying those identifiers that are visible from the outside. All other identifiers will remain local to the module. For global values, their types must be given by hand. The interface is contained in a file whose name is the module name, with extension `.mli`.

Here is for instance an interface for the `counter` module, that abstracts the type `counter`:

```
(* counter.mli *)
type counter;;        (* an abstract type *)
value new : int -> counter
  and incr : counter -> unit
  and read : counter -> int;;
```

Interfaces must be compiled separately. However, once the interface for module A has been compiled, any module B that uses A can be immediately compiled, even if the implementation of A is not yet compiled or even not yet written. Consider:

```
camlc -c counter.mli
camlc -c main.ml
camlc -c counter.ml
camlc -o main counter.zo main.zo
```

The implementation `main.ml` could be compiled before `counter.ml`. The only requirement for compiling `main.ml` is the existence of `counter.zi`, the compiled interface of the `counter` module.

Exercises

Exercise 11.1 *Complete the* `count` *command: it should be able to operate on several files, given on the command line. Hint:* `sys__command_line` *is an array of strings, containing the command-line arguments to the process.*

Part III

A complete example

Chapter 12

ASL: A Small Language

We present in this chapter a simple language: ASL (A Small Language). This language is basically the λ-calculus (the purely functional kernel of Caml) enriched with a conditional construct. The conditional must be a special construct, because our language will be submitted to call-by-value: thus, the conditional cannot be a function.

ASL programs are built up from numbers, variables, functional expressions (λ-abstractions), applications and conditionals. An ASL program consists of a global declaration of an identifier getting bound to the value of an expression. The primitive functions that are available are equality between numbers and arithmetic binary operations. The concrete syntax of ASL expressions can be described (ambiguously) as:

```
Expr ::= INT
       | IDENT
       | "if" Expr "then" Expr "else" Expr "fi"
       | "(" Expr ")"
       | "\" IDENT "." Expr
```

and the syntax of declarations is given as:

```
Decl ::= "let" IDENT "be" Expr ";"
       | Expr ";"
```

Arithmetic binary operations will be written in prefix position and will belong to the class `IDENT`. The `\` symbol will play the role of the Caml keyword `function`.

We start by defining the abstract syntax of ASL expressions and of ASL toplevel phrases. Then we define a parser in order to produce abstract syntax trees from the concrete syntax of ASL programs.

12.1 ASL abstract syntax trees

We encode variable names by numbers. These numbers represent the *binding depth* of variables. For instance, the function of `x` returning `x` (the ASL identity function) will be represented as:

```
Abs("x", Var 1)
```

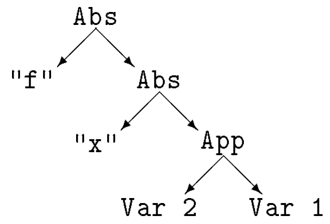
And the ASL application function which would be written in Caml:

```
(function f -> (function x -> f(x)))
```

would be represented as:

```
Abs("f", Abs("x", App(Var 2, Var 1)))
```

and should be viewed as the tree:



`Var n` should be read as "an occurrence of the variable bound by the `n`th abstraction node encountered when going toward the root of the abstract syntax tree". In our example, when going from `Var 2` to the root, the 2nd abstraction node we encounter introduces the `"f"` variable.

The numbers encoding variables in abstract syntax trees of functional expressions are called "De Bruijn[1] numbers". The characters that we attach to abstraction nodes simply serve as documentation: they will not be used by any of the semantic analyses that we will perform on the trees. The type of ASL abstract syntax trees is defined by:

```
#type asl = Const of int
#         | Var of int
#         | Cond of asl * asl * asl
#         | App of asl * asl
#         | Abs of string * asl
#
#and top_asl = Decl of string * asl;;
Type asl defined.
Type top_asl defined.
```

12.2 Parsing ASL programs

Now we come to the problem of defining a concrete syntax for ASL programs and declarations.

The choice of the concrete aspect of the programs is simply a matter of taste. The one we choose here is close to the syntax of λ-calculus (except that we will use the *backslash* character because there is no "λ" on our keyboards). We will use the *curried* versions of equality and arithmetic functions. We will also use a *prefix* notation (à la Lisp) for their application. We will write "`+ (+ 1 2) 3`" instead of "`(1+2)+3`". The "`if` e_1 `then` e_2 `else` e_3" construct will be written "`if` e_1 `then` e_2 `else` e_3 `fi`", and will return the `then` part when e_1 is different from 0 (0 acts thus as falsity in ASL conditionals).

[1] They have been proposed by N.G. De Bruijn in [10] in order to facilitate the mechanical treatment of λ-calculus terms.

12.2.1 Lexical analysis

The concrete aspect of ASL programs will be either declarations of the form:

`let` *identifier* `be` *expression*;

or:

expression;

which will be understood as:

`let it be` *expression*;

The tokens produced by the lexical analyzer will represent the keywords `let`, `be`, `if` and `else`, the `\` binder, the dot, parentheses, integers, identifiers, arithmetic operations and terminating semicolons. We reuse here most of the code that we developed in chapter 10 or in the answers to its exercises.

Skipping blank spaces:

```
#let rec spaces = function
#  [< '` `|`\t`|`\n`; spaces _ >] -> ()
#| [< >] -> ();;
spaces : char stream -> unit = <fun>
```

The type of tokens is given by:

```
#type token = LET | BE | LAMBDA | DOT | LPAR | RPAR
#           | IF | THEN | ELSE | FI | SEMIC
#           | INT of int | IDENT of string;;
Type token defined.
```

Integers:

```
#let int_of_digit = function
#  `0`..`9` as c -> (int_of_char c) - (int_of_char `0`)
#| _ -> raise (Failure "not a digit");;
int_of_digit : char -> int = <fun>

#let rec integer n = function
#  [< ' `0`..`9` as c; (integer (10*n + int_of_digit c)) r >] -> r
#| [< >] -> INT n;;
integer : int -> char stream -> token = <fun>
```

We restrict ASL identifiers to be composed of lowercase letters, the eight first being significative. An explanation about the `ident` function can be found in the chapter dedicated to the answers to exercises (chapter 17). The function given here is slightly different and tests its result in order to see wether it is a keyword (`let`, `be`, ...) or not:

```
#let ident_buf = make_string 8 ` `;;
ident_buf : string = "        "

#let rec ident len = function
```

```
#  [< ' `a`..`z` as c;
#     (if len >= 8 then ident len
#      else begin
#            set_nth_char ident_buf len c;
#            ident (succ len)
#           end) s >] -> s
#| [< >] -> (match sub_string ident_buf 0 len
#            with "let" -> LET
#               | "be" -> BE
#               | "if" -> IF
#               | "then" -> THEN
#               | "else" -> ELSE
#               | "fi" -> FI
#               | s -> IDENT s);;
ident : int -> char stream -> token = <fun>
```

A reasonable lexical analyzer would use a hash table to recognize keywords faster.

Primitive operations are recognized by the following function, which also detects illegal operators and ends of input:

```
#let oper = function
#  [< '`+`|`-`|`*`|`/`|`=` as c >] -> IDENT(make_string 1 c)
#| [< 'c >] -> prerr_string "Illegal character: ";
#              prerr_endline (char_for_read c);
#              raise (Failure "ASL parsing")
#| [< >] -> prerr_endline "Unexpected end of input";
#           raise (Failure "ASL parsing");;
oper : char stream -> token = <fun>
```

The lexical analyzer has the same structure as the one given in chapter 10 except that leading blanks are skipped.

```
#let rec lexer str = spaces str;
#match str with
#  [< '`(`; spaces _ >]  -> [< 'LPAR; lexer str >]
#| [< '`)`; spaces _ >]  -> [< 'RPAR; lexer str >]
#| [< '`\\`; spaces _ >] -> [< 'LAMBDA; lexer str >]
#| [< '`.`; spaces _ >]  -> [< 'DOT; lexer str >]
#| [< '`;`; spaces _ >]  -> [< 'SEMIC; lexer str >]
#| [< '`0`..`9` as c;
#     (integer (int_of_digit c)) tok;
#      spaces _ >]       -> [< 'tok; lexer str >]
#| [< '`a`..`z` as c;
#     (set_nth_char ident_buf 0 c; ident 1) tok;
#     spaces _ >]        -> [< 'tok; lexer str >]
#| [< oper tok; spaces _ >] -> [< 'tok; lexer str >]
#;;
```

```
lexer : char stream -> token stream = <fun>
```

The lexical analyzer returns a stream of tokens that the parser will receive as argument.

12.2.2 Parsing

The final output of our parser will be abstract syntax trees of type `asl` or `top_asl`. This implies that we will detect unbound identifiers at parse-time. In this case, we will raise the `Unbound` exception defined as:

```
#exception Unbound of string;;
Exception Unbound defined.
```

We also need a function which will compute the binding depths of variables. That function simply looks for the position of the first occurrence of a variable name in a list. It will raise `Unbound` if there is no such occurrence.

```
#let binding_depth s rho =
# let rec bind n = function
#     []  -> raise (Unbound s)
#  | t::l -> if s = t then Var n else bind (n+1) l
#  in bind 1 rho
#;;
binding_depth : string -> string list -> asl = <fun>
```

We also need a global environment, containing names of already bound identifiers. The global environment contains predefined names for the equality and arithmetic functions. We represent the global environment as a reference since each ASL declaration will augment it with a new name.

```
#let init_env =  ["+";"-";"*";"/";"="];;
init_env : string list = ["+"; "-"; "*"; "/"; "="]

#let global_env = ref init_env;;
global_env : string list ref = ref ["+"; "-"; "*"; "/"; "="]
```

We now give a parsing function for ASL programs. Blanks at the beginning of the string are skipped.

```
#let rec top = function
#    [< 'LET; 'IDENT id; 'BE; expression e; 'SEMIC >] -> Decl(id,e)
#  | [< expression e; 'SEMIC >] -> Decl("it",e)
#
#and expression = function
#    [< (expr !global_env) e >] -> e
#
#and expr rho =
#  let rec rest e1 = function
#          [< (atom rho) e2; (rest (App(e1,e2))) e >] -> e
#        | [< >] -> e1
```

```
#  in function
#       [< 'LAMBDA; 'IDENT id; 'DOT; (expr (id::rho)) e >] -> Abs(id,e)
#     | [< (atom rho) e1; (rest e1) e2 >] -> e2
#
#and atom rho = function
#    [< 'IDENT id >] ->
#        (try binding_depth id rho with Unbound s ->
#              print_string "Unbound ASL identifier: ";
#              print_string s; print_newline();
#              raise (Failure "ASL parsing"))
#  | [< 'INT n >] -> Const n
#  | [< 'IF; (expr rho) e1; 'THEN; (expr rho) e2;
#       'ELSE; (expr rho) e3; 'FI >] -> Cond(e1,e2,e3)
#  | [< 'LPAR; (expr rho) e; 'RPAR >] -> e;;
top : token stream -> top_asl = <fun>
expression : token stream -> asl = <fun>
expr : string list -> token stream -> asl = <fun>
atom : string list -> token stream -> asl = <fun>
```

The complete parser that we will use reads a string, converts it into a stream, and produces the token stream that is parsed:

```
#let parse_top s = top(lexer(stream_of_string s));;
parse_top : string -> top_asl = <fun>
```

Let us try our grammar (we do not augment the global environment at each declaration: this will be performed after the semantic treatment of ASL programs). We need to write double \ inside strings, since \ is the string escape character.

```
#parse_top "let f be \\x.x;";;
- : top_asl = Decl ("f", Abs ("x", Var 1))
#parse_top "let x be + 1 ((\\x.x) 2);";;
- : top_asl =
 Decl ("x", App (App (Var 1, Const 1), App (Abs ("x", Var 1), Const 2)))
```

Unbound identifiers and undefined operators are correctly detected:

```
#parse_top "let y be g 3;";;
Unbound ASL identifier: g
Uncaught exception: Failure "ASL parsing"
#parse_top "f (if 0 then + else - fi) 2 3;";;
Unbound ASL identifier: f
Uncaught exception: Failure "ASL parsing"
#parse_top "^ x y;";;
Illegal character: ^
Uncaught exception: Failure "ASL parsing"
```

Chapter 13

Untyped semantics of ASL programs

In this section, we give a semantic treatment of ASL programs. We will use *dynamic typechecking*, i.e. we will test the type correctness of programs during their interpretation.

13.1 Semantic values

We need a type for ASL semantic values (representing results of computations). A semantic value will be either an integer, or a Caml functional value from ASL values to ASL values.

```
#type semval = Constval of int
#            | Funval of (semval -> semval);;
Type semval defined.
```

We now define two exceptions. The first one will be used when we encounter an ill-typed program and will represent run-time type errors. The other one is helpful for debugging: it will be raised when our interpreter (semantic function) goes into an illegal situation.

The following two exceptions will be raised in case of run-time ASL type error, and in case of bug of our semantic treatment:

```
#exception Illtyped;;
Exception Illtyped defined.
```

```
#exception SemantBug of string;;
Exception SemantBug defined.
```

We must give a semantic value to our basic functions (equality and arithmetic operations). The next function transforms a Caml function into an ASL value.

```
#let init_semantics caml_fun =
#    Funval
#      (function Constval n ->
#         Funval(function Constval m -> Constval(caml_fun n m)
#                       | _ -> raise Illtyped)
#              | _ -> raise Illtyped);;
init_semantics : (int -> int -> int) -> semval = <fun>
```

Now, associate a Caml Light function to each ASL predefined function:

```
#let caml_function = function
#    "+" -> prefix +
#  | "-" -> prefix -
#  | "*" -> prefix *
#  | "/" -> prefix /
#  | "=" -> (fun n m -> if n=m then 1 else 0)
#  | s -> raise (SemantBug "Unknown primitive");;
caml_function : string -> int -> int -> int = <fun>
```

In the same way as, for parsing, we needed a global environment from which the binding depth of identifiers was computed, we need a semantic environment from which the interpreter will fetch the value represented by identifiers. The global semantic environment will be a reference on the list of predefined ASL values.

```
#let init_sem =  map (fun x -> init_semantics(caml_function x))
#                    init_env;;
init_sem : semval list =
 [Funval <fun>; Funval <fun>; Funval <fun>; Funval <fun>; Funval <fun>]

#let global_sem = ref init_sem;;
global_sem : semval list ref =
 ref [Funval <fun>; Funval <fun>; Funval <fun>; Funval <fun>; Funval <fun>]
```

13.2 Semantic functions

The semantic function is the interpreter itself. There is one for expressions and one for declarations. The one for expressions computes the value of an ASL expression from an environment `rho`. The environment will contain the values of globally defined ASL values or of temporary ASL values. It is organized as a list, and the numbers representing variable occurrences will be used as indices into the environment.

```
#let rec nth n = function
#     []  -> raise (Failure "nth")
#  | x::l -> if n=1 then x else nth (n-1) l;;
nth : int -> 'a list -> 'a = <fun>

#let rec semant rho =
#  let rec sem = function
#      Const n -> Constval n
#    | Var(n) -> nth n rho
#    | Cond(e1,e2,e3) ->
#        (match sem e1 with Constval 0 -> sem e3
#                         | Constval n -> sem e2
#                         | _ -> raise Illtyped)
#    | Abs(_,e') -> Funval(fun x -> semant (x::rho) e')
```

```
#    | App(e1,e2) -> (match sem e1
#                     with Funval(f) -> f (sem e2)
#                        | _ -> raise Illtyped)
#  in sem
#;;
semant : semval list -> asl -> semval = <fun>
```

The main function must be able to treat an ASL declaration, evaluate it, and update the global environments (`global_env` and `global_sem`).

```
#let semantics = function Decl(s,e) ->
#    let result = semant !global_sem e
#    in global_env := s::!global_env;
#       global_sem := result::!global_sem;
#       print_string "ASL Value of ";
#       print_string s;
#       print_string " is ";
#       (match result with
#         Constval n -> print_int n
#       | Funval f -> print_string "<fun>");
#       print_newline();;
semantics : top_asl -> unit = <fun>
```

13.3 Examples

```
#semantics (parse_top "let f be \\x. + x 1;");;
ASL Value of f is <fun>
- : unit = ()

#semantics (parse_top "let i be \\x. x;");;
ASL Value of i is <fun>
- : unit = ()

#semantics (parse_top "let x be i (f 2);");;
ASL Value of x is 3
- : unit = ()

#semantics (parse_top "let y be if x then (\\x.x) else 2 fi 0;");;
ASL Value of y is 0
- : unit = ()
```

Chapter 14

Encoding recursion

14.1 Fixpoint combinators

We have seen that we do not have recursion in ASL. However, it is possible to encode recursion by defining a *fixpoint combinator*. A fixpoint combinator is a function F such that:

$$F\ M \text{ is equivalent to } M\ (F\ M) \text{ modulo the evaluation rules.}$$

for any expression M. A consequence of the equivalence given above is that fixpoint combinators can encode recursion. Let us note $M \equiv N$ if expressions M and N are equivalent modulo the evaluation rules. Then, consider `ffact` to be the functional obtained from the body of the factorial function by abstracting (i.e. using as a parameter) the `fact` identifier, and `fix` an arbitrary fixpoint combinator. We have:

`ffact` is `\fact.(\n. if = n 0 then 1 else * n (fact (- n 1)) fi)`

Now, let us consider the expression $E =$ (`fix ffact`) `3`. Using our intuition about the evaluation rules, and the definition of a fixpoint combinator, we obtain:
$E \equiv$ `ffact (fix ffact) 3`
Replacing `ffact` by its definition, we obtain:
$E \equiv$ `(\fact.(\n. if = n 0 then 1 else * n (fact (- n 1)) fi)) (fix ffact) 3`
We can now pass the two arguments to the first abstraction, instantiating `fact` and `n` respectively to `fix ffact` and `3`:
$E \equiv$ `if = 3 0 then 1 else * 3 (fix ffact (- 3 1)) fi`
We can now reduce the conditional into its `else` branch:
$E \equiv$ `* 3 (fix ffact (- 3 1))`
Continuing this way, we eventually compute:
$E \equiv$ `* 3 (* 2 (* 1 1))` $\equiv$ `6`

This is the expected behavior of the factorial function. Given an appropriate fixpoint combinator `fix`, we could define the factorial function as `fix ffact`, where `ffact` is the expression above.

Unfortunately, when using call-by-value, any application of a fixpoint combinator F such that:

$$F\ M \text{ evaluates to } M\ (F\ M)$$

leads to non-termination of the evaluation (because evaluation of $(F\ M)$ leads to evaluating $(M\ (F\ M))$, and thus $(F\ M)$ again).

We will use the Z fixpoint combinator defined by:

$$Z = \lambda f.((\lambda x.\ f\ (\lambda y.\ (x\ x)\ y))(\lambda x.\ f\ (\lambda y.\ (x\ x)\ y)))$$

The fixpoint combinator Z has the particularity of being usable under call-by-value evaluation regime (in order to check that fact, it is necessary to know the evaluation rules of λ-calculus). Since the name `z` looks more like an ordinary parameter name, we will call `fix` the ASL expression corresponding to the Z fixpoint combinator.

```
#semantics (parse_top
#        "let fix be \\f.((\\x.f(\\y.(x x) y))(\\x.f(\\y.(x x) y)));");;
ASL Value of fix is <fun>
- : unit = ()
```

We are now able to define the ASL factorial function:

```
#semantics (parse_top
#        "let fact be fix (\\f.(\\n. if = n 0 then 1
#                                    else * n (f (- n 1)) fi));");;
ASL Value of fact is <fun>
- : unit = ()

#semantics (parse_top "fact 8;");;
ASL Value of it is 40320
- : unit = ()
```

and the ASL Fibonacci function:

```
#semantics (parse_top
#        "let fib be fix (\\f.(\\n. if = n 1 then 1
#                                   else if = n 2 then 1
#                                        else + (f (- n 1)) (f (- n 2)) fi fi));");;
ASL Value of fib is <fun>
- : unit = ()

#semantics (parse_top "fib 9;");;
ASL Value of it is 34
- : unit = ()
```

14.2 Recursion as a primitive construct

Of course, in a more realistic prototype, we would extend the concrete and abstract syntaxes of ASL in order to support recursion as a primitive construct. We do not do it here because we want to keep ASL simple. This is an interesting non trivial exercise!

Chapter 15

Static typing, polymorphism and type synthesis

We now want to perform static typechecking of ASL programs, that is, to complete typechecking *before* evaluation, making run-time type tests unnecessary during evaluation of ASL programs.

Furthermore, we want to have *polymorphism* (i.e. allow the identity function, for example, to be applicable to any kind of data).

Type synthesis may be seen as a game. When learning a game, we must:

- learn the rules (what is allowed, and what is forbidden);
- learn a winning strategy.

In type synthesis, the rules of the game are called a *type system*, and the winning strategy is the typechecking algorithm.

In the following sections, we give the ASL type system, the algorithm and an implementation of that algorithm. Most of this presentation is borrowed from [7].

15.1 The type system

We study in this section a type system for the ASL language. Then, we present an algorithm performing the type synthesis of ASL programs, and its Caml Light implementation. Because of subtle aspects of the notation used (inference rules), and since some important mathematical notions, such as unification of first-order terms, are not presented here, this chapter may seem obscure at first reading.

The type system we will consider for ASL has been first given by Milner [27] for a subset of the ML language (in fact, a superset of λ-calculus). A *type* is either:

- the type Number;
- or a type variable (α, β, ...);
- or $\tau_1 \rightarrow \tau_2$, where τ_1 and τ_2 are types.

In a type, a type variable is an *unknown*, i.e. a type that we are computing. We will use τ, τ', τ_1, ..., as *metavariables*[1] representing types. This notation is important: we shall use other greek letters to denote other notions in the following sections.

Example $(\alpha \rightarrow \text{Number}) \rightarrow \beta \rightarrow \beta$ is a type.
□

A *type scheme*, is a type where some variables are distinguished as being *generic*. We can represent type schemes by:

$$\forall \alpha_1, \ldots, \alpha_n . \tau \text{ where } \tau \text{ is a type.}$$

Example $\forall \alpha.(\alpha \rightarrow \text{Number}) \rightarrow \beta \rightarrow \beta$ and $(\alpha \rightarrow \text{Number}) \rightarrow \beta \rightarrow \beta$ are type schemes.
□

We will use σ, σ', σ_1, ..., as metavariables representing type schemes. We may also write type schemes as $\forall \vec{\alpha}.\tau$. In this case, $\vec{\alpha}$ represent a (possibly empty) set of generic type variables. When the set of generic variables is empty, we write $\forall.\tau$ or simply τ.

We will write $FV(\sigma)$ for the set of *unknowns* occurring in the type scheme σ. Unknowns are also called *free variables* (they are not bound by a $\forall$ quantifier).

We also write $BV(\sigma)$ (*bound type variables of* σ) for the set of type variables occurring in σ which are not free (i.e. the set of variables universally quantified). Bound type variables are also said to be *generic*.

Example If σ denotes the type scheme $\forall \alpha.(\alpha \rightarrow \text{Number}) \rightarrow \beta \rightarrow \beta$, then we have:

$$FV(\sigma) = \{\beta\}$$

and

$$BV(\sigma) = \{\alpha\}$$

□

A *substitution instance* σ' of a type scheme σ is the type scheme $S(\sigma)$ where S is a substitution of types for *free* type variables appearing in σ. When applying a substitution to a type scheme, a renaming of some bound type variables may become necessary, in order to avoid the capture of a free type variable by a quantifier.

Example

- If σ denotes $\forall \beta.(\beta \rightarrow \alpha) \rightarrow \alpha$ and σ' is $\forall \beta.(\beta \rightarrow (\gamma \rightarrow \gamma)) \rightarrow (\gamma \rightarrow \gamma)$, then σ' is a substitution instance of σ because $\sigma' = S(\sigma)$ where $S = \{\alpha \leftarrow (\gamma \rightarrow \gamma)\}$, i.e. S substitutes the type $\gamma \rightarrow \gamma$ for the variable α.

- If σ denotes $\forall \beta.(\beta \rightarrow \alpha) \rightarrow \alpha$ and σ' is $\forall \delta.(\delta \rightarrow (\beta \rightarrow \beta)) \rightarrow (\beta \rightarrow \beta)$, then σ' is a substitution instance of σ because $\sigma' = S(\sigma)$ where $S = \{\alpha \leftarrow (\beta \rightarrow \beta)\}$. In this case, the renaming of β into δ was necessary: we did not want the variable β introduced by S to be captured by the universal quantification $\forall \beta$.

[1] A metavariable should not be confused with a *variable* or a *type variable*.

□

The type scheme $\sigma' = \forall \beta_1 \ldots \beta_m . \tau'$ is said to be a *generic instance* of $\sigma = \forall \alpha_1 \ldots \alpha_n . \tau$ if there exists a substitution S such that:

- the domain of S is included in $\{\alpha_1, \ldots, \alpha_n\}$;
- $\tau' = S(\tau)$;
- no β_i occurs free in σ.

In other words, a generic instance of a type scheme is obtained by giving more precise values to some generic variables, and (possibly) quantifying some of the new type variables introduced.

Example If $\sigma = \forall \beta.(\beta \to \alpha) \to \alpha$, then $\sigma' = \forall \gamma.((\gamma \to \gamma) \to \alpha) \to \alpha$ is a generic instance of σ. We changed β into $(\gamma \to \gamma)$, and we universally quantified on the newly introduced type variable γ.
□

We express this type system by means of *inference rules*. An inference rule is written as a fraction:

- the numerator is called the *premisses*;
- the denominator is called the *conclusion*.

An inference rule:

$$\frac{P_1 \ \ldots \ P_n}{C}$$

may be read in two ways:

- **"If** P_1, **…and** P_n, **then** C".
- **"In order to prove** C, **it is sufficient to prove** P_1, **…and** P_n".

An inference rule may have no premise: such a rule will be called an *axiom*. A complete proof will be represented by a *proof tree* of the following form:

$$\begin{array}{c} \dfrac{P_1^m}{} \ \cdots \qquad \cdots \ \dfrac{P_l^k}{} \\ \vdots \\ \underline{\qquad} \ \cdots \ \underline{\qquad} \\ \dfrac{P_1^1 \ \ldots \ P_n^1}{C} \end{array}$$

where the leaves of the tree (P_1^m, …, P_l^k) are instances of axioms.

In the premisses and the conclusions appear *judgements* having the form:

$$\Gamma \vdash e : \sigma$$

Such a judgement should be read as "under the typing environment Γ, the expression e has type scheme σ". Typing environments are sets of *typing hypotheses* of the form $x : \sigma$ where x is an identifier name and σ is a type scheme: typing environments give types to the variables occurring free (i.e. unbound) in the expression.

When typing λ-calculus terms, the typing environment is managed as a *stack* (because identifiers possess local scopes). We represent that fact in the presentation of the type system by *removing* the typing hypothesis concerning an identifier name x (if such a typing hypothesis exists) before adding a new typing hypothesis concerning x.

We write $\Gamma - \Gamma(x)$ for the set ot typing hypotheses obtained from Γ by removing the typing hypothesis concerning x (if it exists).

Any numeric constant is of type Number:

$$\frac{}{\Gamma \vdash \texttt{Const}\ \ n : \text{Number}} \qquad \text{(NUM)}$$

We obtain type schemes for variables from the typing environment Γ:

$$\frac{}{\Gamma \cup \{x : \sigma\} \vdash \texttt{Var}\ \ x : \sigma} \qquad \text{(TAUT)}$$

It is possible to instantiate type schemes. The "GenInstance" relation represents generic instantiation.

$$\frac{\Gamma \vdash e : \sigma \qquad \sigma' = \text{GenInstance}(\sigma)}{\Gamma \vdash e : \sigma'} \qquad \text{(INST)}$$

It is possible to generalize type schemes with respect to variables that do not occur free in the set of hypotheses:

$$\frac{\Gamma \vdash e : \sigma \qquad \alpha \notin FV(\Gamma)}{\Gamma \vdash e : \forall \alpha . \sigma} \qquad \text{(GEN)}$$

Typing a conditional:

$$\frac{\Gamma \vdash e_1 : \text{Number} \qquad \Gamma \vdash e_2 : \tau \qquad \Gamma \vdash e_3 : \tau}{\Gamma \vdash (\texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3\ \texttt{fi}) : \tau} \qquad \text{(IF)}$$

Typing an application:

$$\frac{\Gamma \vdash e_1 : \tau \rightarrow \tau' \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash (e_1\ e_2) : \tau'} \qquad \text{(APP)}$$

Typing an abstraction:

$$\frac{(\Gamma - \Gamma(x)) \cup \{x : \tau\} \vdash e : \tau'}{\Gamma \vdash (\lambda x\ \ .e) : \tau \rightarrow \tau'} \qquad \text{(ABS)}$$

The special rule below is the one that introduces polymorphism: this corresponds to the ML `let` construct.

$$\frac{\Gamma \vdash e : \sigma \qquad (\Gamma - \Gamma(x)) \cup \{x : \sigma\} \vdash e' : \tau}{\Gamma \vdash (\lambda x\ \ .e')\ e : \tau} \qquad \text{(LET)}$$

This type system has been proven to be *semantically sound*, i.e. the semantic value of a well-typed expression (an expression admitting a type) cannot be an *error value* due to a type error. This is usually expressed as:

> Well-typed programs cannot go wrong.

This fact implies that a clever compiler may produce code without any dynamic type test for a well-typed expression.

Example Let us check, using the set of rules above, that the following is true:

$$\emptyset \vdash \texttt{let f} = \lambda\texttt{x.x in f f} : \beta \to \beta$$

In order to do so, we will use the equivalence between the `let` construct and an application of an immediate abstraction (i.e. an expression having the following shape: $(\lambda v.M)N$. The (LET) rule will be crucial: without it, we could not check the judgement above.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\{\texttt{x} : \alpha\} \vdash \texttt{x} : \alpha}\,{\scriptstyle(\mathrm{TAUT})}}{\emptyset \vdash (\lambda\texttt{x.x}) : \alpha \to \alpha}\,{\scriptstyle(\mathrm{ABS})}}{\emptyset \vdash (\lambda\texttt{x.x}) : \forall\alpha.\alpha \to \alpha}\,{\scriptstyle(\mathrm{GEN})} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma \vdash \texttt{f} : \forall\alpha.\alpha \to \alpha}\,{\scriptstyle(\mathrm{TAUT})}}{\Gamma \vdash \texttt{f} : (\beta \to \beta) \to (\beta \to \beta)}\,{\scriptstyle(\mathrm{INST})} \quad \dfrac{\dfrac{}{\Gamma \vdash \texttt{f} : \forall\alpha.\alpha \to \alpha}\,{\scriptstyle(\mathrm{TAUT})}}{\Gamma \vdash \texttt{f} : \beta \to \beta}\,{\scriptstyle(\mathrm{INST})}}{\Gamma = \{\texttt{f} : \forall\alpha.\alpha \to \alpha\} \vdash \texttt{f f} : \beta \to \beta}\,{\scriptstyle(\mathrm{APP})}}{\emptyset \vdash \texttt{let f} = \lambda\texttt{x.x in f f} : \beta \to \beta}\,{\scriptstyle(\mathrm{LET})}$$

□

This type system does not tell us how to find the best type for an expression. But what is the best type for an expression? It must be such that any other possible type for that expression is more specific; in other words, the best type is the *most general*.

15.2 The algorithm

How do we find the most general type for an expression of our language? The problem with the set of rules above, is that we could instantiate and generalize types at any time, introducing type schemes, while the most important rules (application and abstraction) used only types.

Let us write a new set of inference rules that we will read as an algorithm (close to a Prolog program):
Any numeric constant is of type Number:

$$\frac{}{\Gamma \vdash \texttt{Const}\ \ n : \mathrm{Number}} \qquad (\mathrm{NUM})$$

The types of identifiers are obtained by taking generic instances of type schemes appearing in the typing environment. These generic instances will be *types* and not type schemes: this restriction appears in the rule below, where the type τ is expected to be a generic instance of the type scheme σ.

As it is presented (belonging to a deduction system), the following rule will have to anticipate the effect of the equality constraints between types in the other rules (multiple occurrences of a type metavariable), when choosing the instance τ.

$$\frac{\tau = \mathrm{GenInstance}(\sigma)}{\Gamma \cup \{x : \sigma\} \vdash \texttt{Var}\ \ x : \tau} \qquad (\mathrm{INST})$$

When we read this set of inference rules as an algorithm, the (INST) rule will be implemented by:

1. taking as τ the "most general generic instance" of σ that is a type (the rule requires τ to be a type and not a type scheme),

2. making τ more specific by *unification* [32] when encountering equality constraints.

Typing a conditional requires only the test part to be of type Number, and both alternatives to be of the same type τ. This is an example of equality constraint between the types of two expressions.

$$\frac{\Gamma \vdash e_1 : \text{Number} \qquad \Gamma \vdash e_2 : \tau \qquad \Gamma \vdash e_3 : \tau}{\Gamma \vdash (\texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \texttt{ fi}) : \tau} \qquad \text{(COND)}$$

Typing an application produces also equality constraints that are to be solved by unification:

$$\frac{\Gamma \vdash e_1 : \tau \to \tau' \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash (e_1 \ e_2) : \tau'} \qquad \text{(APP)}$$

Typing an abstraction "pushes" a typing hypothesis for the abstracted identifier: unification will make it more precise during the typing of the abstraction body:

$$\frac{(\Gamma - \Gamma(x)) \cup \{x : \forall.\tau\} \vdash e : \tau'}{\Gamma \vdash (\lambda x \ .e) : \tau \to \tau'} \qquad \text{(ABS)}$$

Typing a `let` construct involves a generalization step: we generalize as much as possible.

$$\frac{\Gamma \vdash e : \tau' \qquad \{\alpha_1, \ldots, \alpha_n\} = FV(\tau') - FV(\Gamma) \qquad (\Gamma - \Gamma(x)) \cup \{x : \forall \alpha_1 \ldots \alpha_n.\tau'\} \vdash e' : \tau}{\Gamma \vdash (\lambda x \ .e') \ e : \tau} \qquad \text{(LET)}$$

This set of inference rules represents an algorithm because there is exactly one conclusion for each syntactic ASL construct (giving priority to the (LET) rule over the regular application rule). This set of rules may be read as a Prolog program.

This algorithm has been proven to be:

- *syntactically sound*: when the algorithm succeeds on an expression e and returns a type τ, then $e : \tau$.
- *complete*: if an expression e possesses a type τ, then the algorithm will find a type τ' such that τ is an instance of τ'. The returned type τ' is thus the most general type of e.

Example We compute the type that we simply checked in our last example. What is drawn below is the result of the type synthesis: in fact, we run our algorithm with type variables representing unknowns, modifying the previous applications of the (INST) rule when necessary (i.e. when encountering an equality constraint). This is valid, since it can be proved that the correction of the whole deduction tree is preserved by substitution of types for type variables. In a real implementation of the algorithm, the data structures representing types will be submitted to a unification mechanism.

$$\frac{\dfrac{\dfrac{}{\{\mathtt{x} : \alpha\} \vdash \mathtt{x} : \alpha}\,\scriptstyle\text{(INST)}}{\emptyset \vdash (\lambda\mathtt{x}.\mathtt{x}) : \alpha \to \alpha}\,\scriptstyle\text{(ABS)} \qquad \dfrac{\dfrac{}{\Gamma \vdash \mathtt{f} : (\beta \to \beta) \to (\beta \to \beta)}\,\scriptstyle\text{(INST)} \qquad \dfrac{}{\Gamma \vdash \mathtt{f} : \beta \to \beta}\,\scriptstyle\text{(INST)}}{\Gamma = \{\mathtt{f} : \forall \alpha.\alpha \to \alpha\} \vdash \mathtt{f} \ \mathtt{f} : \beta \to \beta}\,\scriptstyle\text{(APP)}}{\emptyset \vdash \mathtt{let} \ \mathtt{f} = \lambda\mathtt{x}.\mathtt{x} \ \mathtt{in} \ \mathtt{f} \ \mathtt{f} : \beta \to \beta}\,\scriptstyle\text{(LET)}$$

Once again, this expression is not typable without the use of the (LET) rule: an error would occur because of the type equality constraints between all occurrences of a variable bound by a "λ". In an effective implementation, a unification error would occur.

□

We may notice from the example above that the algorithm is *syntax-directed*: since, for a given expression, a type deduction for that expression uses exactly one rule per sub-expression, the deduction possesses the same structure as the expression. We can thus reconstruct the ASL expression from its type deduction tree. From the deduction tree above, if we write upper rules as being "arguments" of the ones below and if we annotate the applications of the (INST) and (ABS) rules by the name of the subject variable, we obtain:

$$\mathrm{LET}_f(\mathrm{ABS}_x(\mathrm{INST}_x),\ \mathrm{APP}(\mathrm{INST}_f,\ \mathrm{INST}_f))$$

This is an illustration of the "types-as-propositions and programs-as-proofs" paradigm, also known as the "Curry-Howard isomorphism" (cf. [14]). In this example, we can see the type of the considered expression as a proposition and the expression itself as the proof, and, indeed, we recognize the expression as the deduction tree.

15.3 The ASL type-synthesizer

We now implement the set of inference rules given above.

We need:

- a Caml representation of ASL types and type schemes,
- a management of type environments,
- a unification procedure,
- a typing algorithm.

15.3.1 Representation of ASL types and type schemes

We first need to define a Caml type for our ASL type data structure:

```
#type asl_type = Unknown
#              | Number
#              | TypeVar of vartype
#              | Arrow of asl_type * asl_type
#and vartype = {Index:int; mutable Value:asl_type}
#and asl_type_scheme = Forall of int list * asl_type ;;
Type asl_type defined.
Type vartype defined.
Type asl_type_scheme defined.
```

The `Unknown` ASL type is not really a type: it is the initial value of fresh ASL type variables. We will consider as abnormal a situation where `Unknown` appears in place of a regular ASL type. In such situations, we will raise the following exception:

```
#exception TypingBug of string;;
Exception TypingBug defined.
```

Type variables are allocated by the `new_vartype` function, and their global counter (a local reference) is reset by `reset_vartypes`.

```
#let new_vartype, reset_vartypes =
#(* Generating and resetting unknowns *)
#    let counter = ref 0
#    in (function () -> counter := !counter + 1;
#                        {Index = !counter; Value = Unknown}),
#       (function () -> counter := 0);;
new_vartype : unit -> vartype = <fun>
reset_vartypes : unit -> unit = <fun>
```

15.3.2 Destructive unification of ASL types

We will need to "shorten" type variables: since they are indirections to ASL types, we need to follow these indirections in order to obtain the type that they represent. For the sake of efficiency, we take advantage of this operation to replace multiple indirections by single indirections (shortening).

```
#let rec shorten t =
#    match t with
#     TypeVar {Index=_; Value=Unknown} -> t
#   | TypeVar ({Index=_;
#                 Value=TypeVar {Index=_;
#                                 Value=Unknown} as tv}) -> tv
#   | TypeVar ({Index=_; Value=TypeVar tv1} as tv2)
#            -> tv2.Value <- tv1.Value; shorten t
#   | TypeVar {Index=_; Value=t'} -> t'
#   | Unknown -> raise (TypingBug "shorten")
#   | t' -> t';;
shorten : asl_type -> asl_type = <fun>
```

An ASL type error will be represented by the following exception:

```
#exception TypeClash of asl_type * asl_type;;
Exception TypeClash defined.
```

We will need unification on ASL types with *occur-check*. The following function implements occur-check:

```
#let occurs {Index=n;Value=_} =
#  let rec occrec = function
#        TypeVar{Index=m;Value=_} -> (n=m)
#      | Number -> false
#      | Arrow(t1,t2) -> (occrec t1) or (occrec t2)
#      | Unknown -> raise (TypingBug "occurs")
#  in occrec
#;;
occurs : vartype -> asl_type -> bool = <fun>
```

The unification function: implements destructive unification. Instead of returning the most general unifier, it returns the unificand of two types (their most general common instance). The two arguments are physically modified in order to represent the same type. The unification function will detect type clashes.

```
#let rec unify (tau1,tau2) =
#  match (shorten tau1, shorten tau2)
#  with (* type variable n and type variable m *)
#       (TypeVar({Index=n; Value=Unknown} as tv1) as t1),
#       (TypeVar({Index=m; Value=Unknown} as tv2) as t2)
#           -> if n <> m then tv1.Value <- t2
#     | (* type t1 and type variable *)
#      t1, (TypeVar ({Index=_;Value=Unknown} as tv) as t2)
#            -> if not(occurs tv t1) then tv.Value <- t1
#               else raise (TypeClash (t1,t2))
#     | (* type variable and type t2 *)
#       (TypeVar ({Index=_;Value=Unknown} as tv) as t1), t2
#            -> if not(occurs tv t2) then tv.Value <- t2
#               else raise (TypeClash (t1,t2))
#     | Number, Number -> ()
#     | Arrow(t1,t2), (Arrow(t'1,t'2) as t)
#            -> unify(t1,t'1); unify(t2,t'2)
#     | (t1,t2) -> raise (TypeClash (t1,t2));;
unify : asl_type * asl_type -> unit = <fun>
```

15.3.3 Representation of typing environments

We use `asl_type_scheme list` as typing environments, and we will use the encoding of variables as indices into the environment.

The initial environment is a list of types `(Number -> (Number -> Number))`, which are the types of the ASL primitive functions.

```
#let init_typing_env =
#    map (function s ->
#            Forall([],Arrow(Number,
#                               Arrow(Number,Number))))
#        init_env;;
```

The global typing environment is initialized to the initial typing environment, and will be updated with the type of each ASL declaration, after they are type-checked.

```
#let global_typing_env = ref init_typing_env;;
```

15.3.4 From types to type schemes: generalization

In order to implement generalization, we will need some functions collecting types variables occurring in ASL types.

The following function computes the list of type variables of its argument.

```
#let vars_of_type tau =
# let rec vars vs = function
#       Number -> vs
#     | TypeVar {Index=n; Value=Unknown}
#                 -> if mem n vs then vs else n::vs
#     | TypeVar {Index=_; Value= t} -> vars vs t
#     | Arrow(t1,t2) -> vars (vars vs t1) t2
#     | Unknown -> raise (TypingBug "vars_of_type")
#  in vars [] tau
#;;
vars_of_type : asl_type -> int list = <fun>
```

The `unknowns_of_type(bv,t)` application returns the list of variables occurring in `t` that do not appear in `bv`. The `subtract` function returns the difference of two lists.

```
#let unknowns_of_type (bv,t) =
#    subtract (vars_of_type t) bv;;
unknowns_of_type : int list * asl_type -> int list = <fun>
```

We need to compute the list of unknowns of a type environment for the generalization process (unknowns belonging to that list cannot become generic). The set of unknowns of a type environment is the union of the unknowns of each type. The `flat` function flattens a list of lists.

```
#let flat = it_list (prefix @) [];;
flat : '_a list list -> '_a list = <fun>

#let unknowns_of_type_env env =
#    flat (map (function Forall(gv,t) -> unknowns_of_type (gv,t)) env);;
unknowns_of_type_env : asl_type_scheme list -> int list = <fun>
```

The generalization of a type is relative to a typing environment. The `make_set` function eliminates duplicates in its list argument.

```
#let rec make_set = function
#     []  -> []
#  | x::l -> if mem x l then make_set l else x :: make_set l;;
make_set : 'a list -> 'a list = <fun>

#let generalise_type (gamma, tau) =
#  let genvars =
#        make_set (subtract (vars_of_type tau)
#                           (unknowns_of_type_env gamma))
#  in Forall(genvars, tau)
#;;
generalise_type : asl_type_scheme list * asl_type -> asl_type_scheme = <fun>
```

15.3.5 From type schemes to types: generic instantiation

The following function returns a generic instance of its type scheme argument. A generic instance is obtained by replacing all generic type variables by new unknowns:

```
#let gen_instance (Forall(gv,tau)) =
#  (* We associate a new unknown to each generic variable *)
#  let unknowns = map (function n -> n, TypeVar(new_vartype())) gv in
#  let rec ginstance = function
#        (TypeVar {Index=n; Value=Unknown} as t) ->
#                    (try assoc n unknowns
#                     with Not_found -> t)
#      | TypeVar {Index=_; Value= t} -> ginstance t
#      | Number -> Number
#      | Arrow(t1,t2) -> Arrow(ginstance t1, ginstance t2)
#      | Unknown -> raise (TypingBug "gen_instance")
#  in ginstance tau
#;;
gen_instance : asl_type_scheme -> asl_type = <fun>
```

15.3.6 The ASL type synthesizer

The type synthesizer is the `asl_typing_expr` function. Each of its match cases corresponds to an inference rule given above.

```
#let rec asl_typing_expr gamma =
#  let rec type_rec = function
#      Const _ -> Number
#    | Var n ->
#        let sigma =
#          try nth n gamma
#          with Failure _ -> raise (TypingBug "Unbound")
#        in gen_instance sigma
#    | Cond (e1,e2,e3) ->
#        unify(Number, type_rec e1);
#        let t2 = type_rec e2 and t3 = type_rec e3
#        in unify(t2, t3); t3
#    | App((Abs(x,e2) as f), e1) -> (* LET case *)
#        let t1 = type_rec e1 in
#          let sigma = generalise_type (gamma,t1)
#        in asl_typing_expr (sigma::gamma) e2
#    | App(e1,e2) ->
#        let u = TypeVar(new_vartype())
#        in unify(type_rec e1,Arrow(type_rec e2,u)); u
#    | Abs(x,e) ->
#        let u = TypeVar(new_vartype()) in
```

```
#        let s = Forall([],u)
#        in Arrow(u,asl_typing_expr (s::gamma) e)
#  in type_rec;;
asl_typing_expr : asl_type_scheme list -> asl -> asl_type = <fun>
```

15.3.7 Typing, trapping type clashes and printing ASL types

Now, we define some auxiliary functions in order to build a "good-looking" type synthesizer. First of all, a printing routine for ASL type schemes is defined (using a function `tvar_name` which computes a decent name for type variables).

```
#let tvar_name n =
# (* Computes a name "'a", ... for type variables, *)
# (* given an integer n representing the position  *)
# (* of the type variable in the list of generic   *)
# (* type variables                                *)
# let rec name_of n =
#    let q,r = (n / 26), (n mod 26) in
#    let s = make_string 1 (char_of_int (96+r)) in
#    if q=0 then s else (name_of q)^s
# in "'"^(name_of n)
#;;
tvar_name : int -> string = <fun>
```

Then a printing function for type schemes.

```
#let print_type_scheme (Forall(gv,t)) =
# (* Prints a type scheme.             *)
# (* Fails when it encounters an unknown *)
# let names = let rec names_of = function
#                   (n,[]) -> []
#                 | (n,(v1::Lv)) -> (tvar_name n)::(names_of (n+1, Lv))
#             in names_of (1,gv) in
# let tvar_names = combine (rev gv, names) in
# let rec print_rec = function
#      TypeVar{Index=n; Value=Unknown} ->
#         let name = try assoc n tvar_names
#                    with Not_found ->
#                          raise (TypingBug "Non generic variable")
#         in print_string name
#    | TypeVar{Index=_;Value=t} -> print_rec t
#    | Number -> print_string "Number"
#    | Arrow(t1,t2) ->
#           print_string "("; print_rec t1;
#           print_string " -> "; print_rec t2;
#           print_string ")"
```

```
#    | Unknown -> raise (TypingBug "print_type_scheme")
# in print_rec t
#;;
Toplevel input:
>                 | (n,(v1::Lv)) -> (tvar_name n)::(names_of (n+1, Lv))
>                             ^^
Warning: the variable Lv starts with an upper case letter in this pattern.
print_type_scheme : asl_type_scheme -> unit = <fun>
```

Now, the main function which resets the type variables counter, calls the type synthesizer, traps ASL type clashes and prints the resulting types. At the end, the global environments are updated.

```
#let typing (Decl(s,e)) =
# reset_vartypes();
# let tau = (* TYPING *)
#     try asl_typing_expr !global_typing_env e
#     with TypeClash(t1,t2) -> (* A typing error *)
#           let vars=vars_of_type(t1)@vars_of_type(t2) in
#           print_string "ASL Type clash between ";
#           print_type_scheme (Forall(vars,t1));
#           print_string " and ";
#           print_type_scheme (Forall(vars,t2));
#           print_newline();
#           raise (Failure "ASL typing") in
# let sigma = generalise_type (!global_typing_env,tau) in
# (* UPDATING ENVIRONMENTS *)
# global_env := s::!global_env;
# global_typing_env := sigma::!global_typing_env;
# reset_vartypes ();
# (* PRINTING RESULTING TYPE *)
# print_string "ASL Type of ";
# print_string s;
# print_string " is ";
# print_type_scheme sigma; print_newline();;
typing : top_asl -> unit = <fun>
```

15.3.8 Typing ASL programs

We reinitialize the parsing environment:

```
#global_env:=init_env; ();;
- : unit = ()
```

Now, let us run some examples through the ASL type checker:

```
#typing (parse_top "let x be 1;");;
ASL Type of x is Number
```

```
- : unit = ()

#typing (parse_top "+ 2 ((\\x.x) 3);");;
ASL Type of it is Number
- : unit = ()

#typing (parse_top "if + 0 1 then 1 else 0 fi;");;
ASL Type of it is Number
- : unit = ()

#typing (parse_top "let id be \\x.x;");;
ASL Type of id is ('a -> 'a)
- : unit = ()

#typing (parse_top "+ (id 1) (id id 2);");;
ASL Type of it is Number
- : unit = ()

#typing (parse_top "let f be (\\x.x x) (\\x.x);");;
ASL Type of f is ('a -> 'a)
- : unit = ()

#typing (parse_top "+ (\\x.x) 1;");;
ASL Type clash between Number and ('a -> 'a)
Uncaught exception: Failure "ASL typing"
```

15.3.9 Typing and recursion

The Z fixpoint combinator does not have a type in Milner's type system:

```
#typing (parse_top
#  "let fix be \\f.((\\x.f(\\z.(x x)z)) (\\x.f(\\z.(x x)z)));");;
ASL Type clash between 'a and ('a -> 'b)
Uncaught exception: Failure "ASL typing"
```

This is because we try to apply `x` to itself, and the type of `x` is not polymorphic. In fact, no fixpoint combinator is typable in ASL. This is why we need a special primitive or syntactic construct in order to express recursivity.

If we want to assign types to recursive programs, we have to predefine the Z fixpoint combinator. Its type scheme should be $\forall\alpha.((\alpha \to \alpha) \to \alpha)$, because we take fixpoints of functions.

```
#global_env := "fix"::init_env;
#global_typing_env:=
#    (Forall([1],
#     Arrow(Arrow(TypeVar{Index=1;Value=Unknown},
#                 TypeVar{Index=1;Value=Unknown}),
#           TypeVar{Index=1;Value=Unknown})))
#   ::init_typing_env;
#();;
- : unit = ()
```

We can now define our favorite functions as:

```
#typing (parse_top
#    "let fact be fix (\\f.(\\n. if = n 0 then 1
#                                 else * n (f (- n 1))
#                                 fi));");;
ASL Type of fact is (Number -> Number)
- : unit = ()

#typing (parse_top "fact 8;");;
ASL Type of it is Number
- : unit = ()

#typing (parse_top
#  "let fib be fix (\\f.(\\n. if = n 1 then 1
#                              else if = n 2 then 1
#                                   else + (f(- n 1)) (f(- n 2))
#                                   fi
#                              fi));");;
ASL Type of fib is (Number -> Number)
- : unit = ()

#typing (parse_top "fib 9;");;
ASL Type of it is Number
- : unit = ()
```

Chapter 16

Compiling ASL to an abstract machine code

In order to fully take advantage of the static typing of ASL programs, we have to:

- either write a new interpreter without type tests (difficult, because we used pattern-matching in order to realize type tests);
- or design an untyped machine and produce code for it.

We choose here the second solution: it will permit us to give some intuition about the compiling process of functional languages, and to describe a typical execution model for (strict) functional languages. The machine that we will use is a simplified version the *Categorical Abstract Machine* (CAM, for short).

We will call CAM our abstract machine, despite its differences with the original CAM. For more informations on the CAM, see [8, 25].

16.1 The Abstract Machine

The execution model is a *stack machine* (i.e. a machine using a stack). In this section, we define in Caml the *states* of the CAM and its instructions.

A state is composed of:

- a *register* (holding values and environments),
- a *program counter*, represented here as a list of instructions whose first element is the current instruction being executed,
- and a *stack* represented as a list of code addresses (instruction lists), values and environments.

The first Caml type that we need is the type for CAM instructions. We will study later the effect of each instruction.

```
#type instruction =
#  Quote of int                (* Integer constants *)
```

```
#| Plus | Minus               (* Arithmetic operations *)
#| Divide | Equal | Times
#| Nth of int                 (* Variable accesses *)
#| Branch of instruction list * instruction list
#                             (* Conditional execution *)
#| Push                       (* Pushes onto the stack *)
#| Swap                       (* Exch. top of stack and register *)
#| Clos of instruction list   (* Builds a closure with the current environment *)
#| Apply                      (* Function application *)
#;;
Type instruction defined.
```

We need a new type for semantic values since instruction lists have now replaced abstract syntax trees. The semantic values are merged in a type `object`. The type `object` behaves as data in a computer memory: we need higher-level information (such as type information) in order to interpret them. Furthermore, some data do not correspond to anything (for example an environment composed of environments represents neither an ASL value nor an intermediate data in a legal computation process).

```
#type object = Constant of int
#            | Closure of object * object
#            | Address of instruction list
#            | Environment of object list
#;;
Type object defined.
```

The type `state` is a product type with mutable components.

```
#type state = {mutable Reg: object;
#              mutable PC: instruction list;
#              mutable Stack: object list}
#;;
Type state defined.
```

Now, we give the *operational semantics* of CAM instructions. The effect of an instruction is to change the state configuration. This is what we describe now with the `step` function. Code executions will be arbitrary iterations of this function.

```
#exception CAMbug of string;;
Exception CAMbug defined.

#exception CAM_End of object;;
Exception CAM_End defined.

#let step state = match state with
#  {Reg=_; PC=Quote(n)::code; Stack=s} ->
#                 state.Reg <- Constant(n); state.PC <- code
#
```

```
#| {Reg=Constant(m); PC=Plus::code; Stack=Constant(n)::s} ->
#               state.Reg <- Constant(n+m); state.Stack <- s;
#               state.PC <- code
#
#| {Reg=Constant(m); PC=Minus::code; Stack=Constant(n)::s} ->
#               state.Reg <- Constant(n-m); state.Stack <- s;
#               state.PC <- code
#
#| {Reg=Constant(m); PC=Times::code; Stack=Constant(n)::s} ->
#               state.Reg <- Constant(n*m); state.Stack <- s;
#               state.PC <- code
#
#| {Reg=Constant(m); PC=Divide::code; Stack=Constant(n)::s} ->
#               state.Reg <- Constant(n/m); state.Stack <- s;
#               state.PC <- code
#
#| {Reg=Constant(m); PC=Equal::code; Stack=Constant(n)::s} ->
#               state.Reg <- Constant(if n=m then 1 else 0);
#               state.Stack <- s; state.PC <- code
#
#| {Reg=Constant(m); PC=Branch(code1,code2)::code; Stack=r::s} ->
#               state.Reg <- r;
#               state.Stack <- Address(code)::s;
#               state.PC <- (if m=0 then code2 else code1)
#
#| {Reg=r; PC=Push::code; Stack=s} ->
#               state.Stack <- r::s; state.PC <- code
#
#| {Reg=r1; PC=Swap::code; Stack=r2::s} ->
#               state.Reg <- r2; state.Stack <- r1::s;
#               state.PC <- code
#
#| {Reg=r; PC=Clos(code1)::code; Stack=s} ->
#               state.Reg <- Closure(Address(code1),r);
#               state.PC <- code
#
#| {Reg=_; PC=[]; Stack=Address(code)::s} ->
#               state.Stack <- s; state.PC <- code
#
#| {Reg=v; PC=Apply::code;
#         Stack=Closure(Address(code1),Environment(e))::s} ->
#               state.Reg <- Environment(v::e);
#               state.Stack <- Address(code)::s;
#               state.PC <- code1
#
```

```
#| {Reg=v; PC=[]; Stack=[]} ->
#                raise (CAM_End v)
#| {Reg=_; PC=(Plus|Minus|Times|Divide|Equal)::code; Stack=_::_} ->
#                raise (CAMbug "IllTyped")
#
#| {Reg=Environment(e); PC=Nth(n)::code; Stack=_} ->
#                state.Reg <- (try nth n e
#                              with Failure _ -> raise (CAMbug "IllTyped"));
#                state.PC <- code
#| _ -> raise (CAMbug "Wrong configuration")
#;;
step : state -> unit = <fun>
```

We may notice that the empty code sequence denotes a (possibly final) *return* instruction.

We could argue that pattern-matching in the Camlstep function implements a kind of dynamic typechecking. In fact, in a concrete (low-level) implementation of the machine (expansion of the CAM instructions in assembly code, for example), these tests would not appear. They are useless since we trust the typechecker and the compiler. So, any execution error in a real implementation comes from a *bug* in one of the above processes and would lead to memory errors or illegal instructions (usually detected by the computer's operating system).

16.2 Compiling ASL programs into CAM code

We give in this section a compiling function taking the abstract syntax tree of an ASL expression and producing CAM code. The compilation scheme is very simple:

- the code of a constant is `Quote`;
- a variable is compiled as an access to the appropriate component of the current environment (`Nth`);
- the code of a conditional expression will save the current environment (`Push`), evaluate the condition part, and, according to the boolean value obtained, select the appropriate code to execute (`Branch`);
- the code of an application will also save the environment on the stack (`Push`), execute the function part of the application, then exchange the functional value and the saved environment (`Swap`), evaluate the argument and, finally, apply the functional value (which is at the top of the stack) to the argument held in the register with the `Apply` instruction;
- the code of an abstraction simply consists in building a closure representing the functional value: the closure is composed of the code of the function and the current environment.

Here is the compiling function:

```
#let rec code_of = function
#  Const(n) -> [Quote(n)]
#| Var n -> [Nth(n)]
```

```
#| Cond(e_test,e_t,e_f) ->
#        Push::(code_of e_test)
#      @[Branch(code_of e_t, code_of e_f)]
#| App(e1,e2) -> Push::(code_of e1)
#              @[Swap]@(code_of e2)
#              @[Apply]
#| Abs(_,e) -> [Clos(code_of e)];;
code_of : asl -> instruction list = <fun>
```

A global environment is needed in order to maintain already defined values. Any CAM execution will start in a state whose register part contains this global environment.

```
#let init_CAM_env =
#  let basic_instruction = function
#        "+" -> Plus
#      | "-" -> Minus
#      | "*" -> Times
#      | "/" -> Divide
#      | "=" -> Equal
#      | s -> raise (CAMbug "Unknown primitive")
#  in map (function s ->
#          Closure(Address[Clos(Push::Nth(2)
#                              ::Swap::Nth(1)
#                              ::[basic_instruction s])],
#                   Environment []))
#        init_env;;
init_CAM_env : object list =
 [Closure (Address [Clos [Push; Nth 2; Swap; Nth 1; Plus]], Environment []);
  Closure (Address [Clos [Push; Nth 2; Swap; Nth 1; Minus]], Environment []);
  Closure (Address [Clos [Push; Nth 2; Swap; Nth 1; Times]], Environment []);
  Closure (Address [Clos [Push; Nth 2; Swap; Nth 1; Divide]], Environment []);
  Closure (Address [Clos [Push; Nth 2; Swap; Nth 1; Equal]], Environment [])]
```

```
#let global_CAM_env = ref init_CAM_env;;
global_CAM_env : object list ref =
 ref
  [Closure (Address [Clos [Push; Nth 2; Swap; Nth 1; Plus]], Environment []);
   Closure (Address [Clos [Push; Nth 2; Swap; Nth 1; Minus]], Environment []);
   Closure (Address [Clos [Push; Nth 2; Swap; Nth 1; Times]], Environment []);
   Closure
    (Address [Clos [Push; Nth 2; Swap; Nth 1; Divide]], Environment []);
   Closure (Address [Clos [Push; Nth 2; Swap; Nth 1; Equal]], Environment [])]
```

As an example, here is the code for some ASL expressions.

```
#code_of (expression(lexer(stream_of_string "1;")));;
- : instruction list = [Quote 1]
```

```
#code_of (expression(lexer(stream_of_string "+ 1 2;")));;
- : instruction list =
 [Push; Push; Nth 6; Swap; Quote 1; Apply; Swap; Quote 2; Apply]
```

```
#code_of (expression(lexer(stream_of_string "(\\x.x) ((\\x.x) 0);")));;
- : instruction list =
 [Push; Clos [Nth 1]; Swap; Push; Clos [Nth 1]; Swap; Quote 0; Apply; Apply]
```

```
#code_of (expression(lexer(stream_of_string
#                "+ 1 (if 0 then 2 else 3 fi);")));;
- : instruction list =
 [Push; Push; Nth 6; Swap; Quote 1; Apply; Swap; Push; Quote 0;
  Branch ([Quote 2], [Quote 3]); Apply]
```

16.3 Execution of CAM code

The main function for executing compiled ASL manages the global environment until execution has succeeded.

```
#let run (Decl(s,e)) =
#  (* TYPING *)
#    reset_vartypes();
#    let tau =
#        try asl_typing_expr !global_typing_env e
#        with TypeClash(t1,t2) ->
#              let vars=vars_of_type(t1) @ vars_of_type(t2) in
#              print_string "ASL Type clash between ";
#              print_type_scheme (Forall(vars,t1));
#              print_string " and ";
#              print_type_scheme (Forall(vars,t2));
#              raise (Failure "ASL typing")
#           | Unbound s -> raise (TypingBug ("Unbound: "^s)) in
#    let sigma = generalise_type (!global_typing_env,tau) in
#  (* PRINTING TYPE INFORMATION *)
#    print_string "ASL Type of ";
#    print_string s; print_string " is ";
#    print_type_scheme sigma; print_newline();
#  (* COMPILING *)
#    let code = code_of e in
#    let state = {Reg=Environment(!global_CAM_env); PC=code; Stack=[]} in
#  (* EXECUTING *)
#    let result = try while true do step state done; state.Reg
#                 with CAM_End v -> v in
#  (* UPDATING ENVIRONMENTS *)
#    global_env := s::!global_env;
#    global_typing_env := sigma::!global_typing_env;
```

```
#    global_CAM_env := result::!global_CAM_env;
#  (* PRINTING RESULT *)
#    (match result
#     with Constant(n) -> print_int n
#        | Closure(_,_) -> print_string "<funval>"
#        | _ -> raise (CAMbug "Wrong state configuration"));
#    print_newline();;
run : top_asl -> unit = <fun>
```

Now, let us run some examples:

```
#(* Reinitializing environments *)
#global_env:=init_env;
#global_typing_env:=init_typing_env;
#global_CAM_env:=init_CAM_env;
#();;
- : unit = ()
```

```
#run (parse_top "1;");;
ASL Type of it is Number
1
- : unit = ()
```

```
#run (parse_top "+ 1 2;");;
ASL Type of it is Number
3
- : unit = ()
```

```
#run (parse_top "(\\f.(\\x.f x)) (\\x. + x 1) 3;");;
ASL Type of it is Number
4
- : unit = ()
```

We may now introduce the Z fixpoint combinator as a predefined function `fix`.

```
#begin
#  global_env:="fix"::init_env;
#  global_typing_env:=
#    (Forall([1],
#            Arrow(Arrow(TypeVar{Index=1;Value=Unknown},
#                        TypeVar{Index=1;Value=Unknown}),
#                  TypeVar{Index=1;Value=Unknown})))
#   ::init_typing_env;
#  global_CAM_env:=
#   (match code_of (expression(lexer(stream_of_string
#          "\\f.((\\x.f(\\z.(x x)z)) (\\x.f(\\z.(x x)z)));")))
#    with [Clos(C)] -> Closure(Address(C), Environment [])
#       | _ -> raise (CAMbug "Wrong code for fix"))
#   ::init_CAM_env
```

```
#end;;
Toplevel input:
>    with [Clos(C)] -> Closure(Address(C), Environment [])
>                ^
Warning: the variable C starts with an upper case letter in this pattern.
- : unit = ()

#run (parse_top
#    "let fact be fix (\\f.(\\n. if = n 0 then 1
#                             else * n (f (- n 1))
#                             fi));");;
ASL Type of fact is (Number -> Number)
<funval>
- : unit = ()

#run (parse_top
#  "let fib be fix (\\f.(\\n. if = n 1 then 1
#                               else if = n 2 then 1
#                                    else + (f(- n 1)) (f(- n 2))
#                                    fi
#                               fi));");;
ASL Type of fib is (Number -> Number)
<funval>
- : unit = ()

#run (parse_top "fact 8;");;
ASL Type of it is Number
40320
- : unit = ()

#run (parse_top "fib 9;");;
ASL Type of it is Number
34
- : unit = ()
```

It is of course possible (and desirable) to introduce recursion by using a specific syntactic construct, special instructions and a dedicated case to the compiling function. See [25] for efficient compilation of recursion, data structures etc.

Exercise 16.1 *Interesting exercises for which we won't give solutions consist in enriching according to your taste the ASL language. Also, building a standalone ASL interpreter is a good exercise in modular programming.*

Chapter 17

Answers to exercises

We give in this chapter one possible solution for each exercise contained in this document. Exercises are referred to by their number and the page where they have been proposed: for example, "2.1, p. 15" refers to the first exercise in chapter 2; this exercise is located on page 15.

3.1, p. 19

The following (anonymous) functions have the required types:

1. ```
   #function f -> (f 2)+1;;
   - : (int -> int) -> int = <fun>
   ```
2. ```
   #function m -> (function n -> n+m+1);;
   - : int -> int -> int = <fun>
   ```
3. ```
   #(function f -> (function m -> f(m+1) / 2));;
   - : (int -> int) -> int -> int = <fun>
   ```

3.2, p. 19

We must first rename `y` to `z`, obtaining:

```
(function x -> (function z -> x+z))
```

and finally:

```
(function y -> (function z -> y+z))
```

Without renaming, we would have obtained:

```
(function y -> (function y -> y+y))
```

which does not denote the same function.
```

3.3, p. 19

We write successively the reduction steps for each expressions, and then we use Caml in order to check the result.

- ```
  let x=1+2 in ((function y -> y+x) x);;
  (function y -> y+(1+2)) (1+2);;
  (function y -> y+(1+2)) 3;;
  3+(1+2);;
  3+3;;
  6;;
  ```
  Caml says:

```
#let x=1+2 in ((function y -> y+x) x);;
- : int = 6
```

- ```
  let x=1+2 in ((function x -> x+x) x);;
  (function x -> x+x) (1+2);;
  3+3;;
  6;;
  ```
  Caml says:

```
#let x=1+2 in ((function x -> x+x) x);;
- : int = 6
```

- ```
  let f1 = function f2 -> (function x -> f2 x)
  in let g = function x -> x+1
     in f1 g 2;;
  let g = function x -> x+1
  in function f2 -> (function x -> f2 x) g 2;;
  (function f2 -> (function x -> f2 x)) (function x -> x+1) 2;;
  (function x -> (function x -> x+1) x)  2;;
  (function x -> x+1) 2;;
  2+1;;
  3;;
  ```
  Caml says:

```
#let f1 = function f2 -> (function x -> f2 x)
#in let g = function x -> x+1
#   in f1 g 2;;
- : int = 3
```

4.1, p. 31

To compute the surface area of a rectangle and the volume of a sphere:
```

```
#let surface_rect len wid = len * wid;;
surface_rect : int -> int -> int = <fun>

#let pi = 4.0 *. atan 1.0;;
pi : float = 3.14159265359

#let volume_sphere r = 4.0 /. 3.0 *. pi *. (power r 3.);;
volume_sphere : float -> float = <fun>
```

4.2, p. 31

In a call-by-value language without conditional construct (and without sum types), all programs involving a recursive definition never terminate.

4.3, p. 31

```
#let rec factorial n = if n=1 then 1 else n*(factorial(n-1));;
factorial : int -> int = <fun>

#factorial 5;;
- : int = 120

#let tail_recursive_factorial n =
#  let rec fact n m = if n=1 then m else fact (n-1) (n*m)
#  in fact n 1;;
tail_recursive_factorial : int -> int = <fun>

#tail_recursive_factorial 5;;
- : int = 120
```

4.4, p. 31

```
#let rec fibonacci n =
#  if n=1 then 1
#         else if n=2 then 1
#                        else fibonacci(n-1) + fibonacci(n-2);;
fibonacci : int -> int = <fun>

#fibonacci 20;;
- : int = 6765
```

4.5, p. 32

```
#let compose f g = function x -> f (g (x));;
compose : ('a -> 'b) -> ('c -> 'a) -> 'c -> 'b = <fun>

#let curry f = function x -> (function y -> f(x,y));;
curry : ('a * 'b -> 'c) -> 'a -> 'b -> 'c = <fun>
```

```
#let uncurry f = function (x,y) -> f x y;;
uncurry : ('a -> 'b -> 'c) -> 'a * 'b -> 'c = <fun>

#uncurry compose;;
- : ('_a -> '_b) * ('_c -> '_a) -> '_c -> '_b = <fun>

#compose curry uncurry;;
- : ('_a -> '_b -> '_c) -> '_a -> '_b -> '_c = <fun>

#compose uncurry curry;;
- : ('_a * '_b -> '_c) -> '_a * '_b -> '_c = <fun>
```

5.1, p. 36

```
#let rec combine =
#  function [],[] -> []
#         | (x1::l1),(x2::l2) -> (x1,x2)::(combine(l1,l2))
#         | _ -> raise (Failure "combine: lists of different length");;
combine : 'a list * 'b list -> ('a * 'b) list = <fun>

#combine ([1;2;3],["a";"b";"c"]);;
- : (int * string) list = [1, "a"; 2, "b"; 3, "c"]

#combine ([1;2;3],["a";"b"]);;
Uncaught exception: Failure "combine: lists of different length"
```

5.2, p. 36

```
#let rec sublists =
#    function [] -> [[]]
#           | x::l -> let sl = sublists l
#                     in sl @ (map (fun l -> x::l) sl);;
sublists : 'a list -> 'a list list = <fun>

#sublists [];;
- : '_a list list = [[]]

#sublists [1;2;3];;
- : int list list = [[]; [3]; [2]; [2; 3]; [1]; [1; 3]; [1; 2]; [1; 2; 3]]

#sublists ["a"];;
- : string list list = [[]; ["a"]]
```

6.1, p. 46

```
#type ('a,'b) btree = Leaf of 'b
#                   | Btree of ('a,'b) node
#and ('a,'b) node = {Op:'a;
```

```
#                    Son1: ('a,'b) btree;
#                    Son2: ('a,'b) btree};;
Type btree defined.
Type node defined.
```

```
#let rec nodes_and_leaves =
#    function Leaf x -> ([],[x])
#           | Btree {Op=x; Son1=s1; Son2=s2} ->
#                let (nodes1,leaves1) = nodes_and_leaves s1
#                and (nodes2,leaves2) = nodes_and_leaves s2
#                in (x::nodes1@nodes2, leaves1@leaves2);;
nodes_and_leaves : ('a, 'b) btree -> 'a list * 'b list = <fun>
```

```
#nodes_and_leaves (Btree {Op="+"; Son1=Leaf 1; Son2=Leaf 2});;
- : string list * int list = ["+"], [1; 2]
```

6.2, p. 46

```
#let rec map_btree f g = function
#      Leaf x -> Leaf (f x)
#    | Btree {Op=op; Son1=s1; Son2=s2}
#               -> Btree {Op=g op; Son1=map_btree f g s1;
#                                  Son2=map_btree f g s2};;
map_btree : ('a -> 'b) -> ('c -> 'd) -> ('c, 'a) btree -> ('d, 'b) btree =
 <fun>
```

6.3, p. 46

We need to give a functional interpretation to `btree` data constructors. We use `f` (resp. `g`) to denote the function associated to the `Leaf` (resp. `Btree`) data constructor, obtaining the following Caml definition:

```
#let rec btree_it f g = function
#      Leaf x -> f x
#    | Btree{Op=op; Son1=s1; Son2=s2}
#              -> g op (btree_it f g s1) (btree_it f g s2)
#;;
btree_it : ('a -> 'b) -> ('c -> 'b -> 'b -> 'b) -> ('c, 'a) btree -> 'b =
 <fun>
```

```
#btree_it (function x -> x)
#         (function "+" -> prefix +
#                 | _ -> raise (Failure "Unknown op"))
#         (Btree {Op="+"; Son1=Leaf 1; Son2=Leaf 2});;
- : int = 3
```

7.1, p. 54

```
#type ('a,'b) lisp_cons = {mutable Car:'a; mutable Cdr:'b};;
Type lisp_cons defined.

#let car p = p.Car
#and cdr p = p.Cdr
#and rplaca p v = p.Car <- v
#and rplacd p v = p.Cdr <- v;;
car : ('a, 'b) lisp_cons -> 'a = <fun>
cdr : ('a, 'b) lisp_cons -> 'b = <fun>
rplaca : ('a, 'b) lisp_cons -> 'a -> unit = <fun>
rplacd : ('a, 'b) lisp_cons -> 'b -> unit = <fun>

#let p = {Car=1; Cdr=true};;
p : (int, bool) lisp_cons = {Car = 1; Cdr = true}

#rplaca p 2;;
- : unit = ()

#p;;
- : (int, bool) lisp_cons = {Car = 2; Cdr = true}
```

7.2, p. 54

```
#let stamp_counter = ref 0;;
stamp_counter : int ref = ref 0

#let stamp () =
#  stamp_counter := 1 + !stamp_counter; !stamp_counter;;
stamp : unit -> int = <fun>

#stamp();;
- : int = 1

#stamp();;
- : int = 2
```

7.3, p. 54

```
#let exchange t i j =
#  let v = t.(i) in vect_assign t i t.(j); vect_assign t j v
#;;
exchange : 'a vect -> int -> int -> unit = <fun>

#let quick_sort t =
#  let rec quick lo hi =
#    if lo < hi
#    then begin
```

```
#          let i = ref lo
#          and j = ref hi
#          and p = t.(hi) in
#          while !i < !j
#           do
#            while !i < hi & t.(!i) <=. p do incr i done;
#            while !j > lo & p <=. t.(!j) do decr j done;
#            if !i < !j then exchange t !i !j
#           done;
#          exchange t hi !i;
#          quick lo (!i - 1);
#          quick (!i + 1) hi
#         end
#     else ()
#  in quick 0 (vect_length t - 1)
#;;
quick_sort : float vect -> unit = <fun>

#let a = [| 2.0; 1.5; 4.0; 0.0; 10.0; 1.0 |];;
a : float vect = [|2.0; 1.5; 4.0; 0.0; 10.0; 1.0|]

#quick_sort a;;
- : unit = ()

#a;;
- : float vect = [|0.0; 1.0; 1.5; 2.0; 4.0; 10.0|]
```

8.1, p. 58

```
#let rec find_succeed f = function
#      [] -> raise (Failure "find_succeed")
#    | x::l -> try f x; x with _ -> find_succeed f l
#;;
find_succeed : ('a -> 'b) -> 'a list -> 'a = <fun>

#let hd = function [] -> raise (Failure "empty") | x::l -> x;;
hd : 'a list -> 'a = <fun>

#find_succeed hd [[];[];[1;2];[3;4]];;
- : int list = [1; 2]
```

8.2, p. 58

```
#let rec map_succeed f = function
#      [] -> []
#    | h::t -> try (f h)::(map_succeed f t)
#              with _ ->  map_succeed f t;;
```

```
map_succeed : ('a -> 'b) -> 'a list -> 'b list = <fun>

#map_succeed hd [[];[1];[2;3];[4;5;6]];;
- : int list = [1; 2; 4]
```

9.1, p. 63

The first function (copy) that we define assumes that its arguments are respectively the input and the output channel. They are assumed to be already opened.

```
#let copy inch outch =
#  (* inch and outch are supposed to be opened channels *)
#  try (* actual copying *)
#      while true
#      do output_char outch (input_char inch)
#      done
#   with End_of_file -> (* Normal termination *)
#             raise End_of_file
#      | sys__Sys_error msg -> (* Abnormal termination *)
#             prerr_endline msg;
#             raise (Failure "cp")
#      | _ -> (* Unknow exception, maybe interruption? *)
#             prerr_endline "Unknown error while copying";
#             raise (Failure "cp")
#;;
copy : in_channel -> out_channel -> unit = <fun>
```

The next function opens channels connected to its filename arguments, and calls copy on these channels. The advantage of dividing the code into two functions is that copy performs the actual work, and can be reused in different applications, while the role of cp is more "administrative" in the sense that it does nothing but opening and closing channels and printing possible error messages.

```
#let cp f1 f2 =
#  (* Opening channels, f1 first, then f2 *)
#  let inch =
#      try open_in f1
#      with sys__Sys_error msg ->
#                prerr_endline (f1^": "^msg);
#                raise (Failure "cp")
#         | _ -> prerr_endline ("Unknown exception while opening "^f1);
#                raise (Failure "cp")
#  in
#  let outch =
#      try open_out f2
#      with sys__Sys_error msg ->
```

```
#                close_in inch;
#                prerr_endline (f2^": "^msg);
#                raise (Failure "cp")
#          | _ -> close_in inch;
#                prerr_endline ("Unknown exception while opening "^f2);
#                raise (Failure "cp")
#  in (* Copying and then closing *)
#     try copy inch outch
#     with End_of_file -> close_in inch; close_out outch
#                        (* close_out flushes *)
#        | exc -> close_in inch; close_out outch; raise exc
#;;
cp : string -> string -> unit = <fun>
```

Let us try cp:

```
#cp "/etc/passwd" "/tmp/foo";;
- : unit = ()
```

```
#cp "/tmp/foo" "/foo";;
/foo: /foo: Permission denied
Uncaught exception: Failure "cp"
```

The last example failed because a regular user is not allowed to write at the root of the file system.

9.2, p. 63

As in the previous exercise, the function count performs the actual counting. It works on an input channel and returns a pair of integers.

```
#let count inch =
#    let chars = ref 0
#    and lines = ref 0 in
#    try
#      while true do
#        let c = input_char inch in
#          chars := !chars + 1;
#          if c = `\n` then lines := !lines + 1 else ()
#      done;
#      (!chars, !lines)
#    with End_of_file -> (!chars, !lines)
#;;
count : in_channel -> int * int = <fun>
```

The function wc opens a channel on its filename argument, calls count and prints the result.

```
#let wc f =
#    let inch =
```

```
#       try open_in f
#       with sys__Sys_error msg ->
#               prerr_endline (f^": "^msg);
#               raise (Failure "wc")
#          | _ -> prerr_endline ("Unknown exception while opening "^f);
#               raise (Failure "wc")
#  in let (chars,lines) = count inch
#     in   print_int chars;
#          print_string " characters, ";
#          print_int lines;
#          print_string " lines.\n"
#;;
wc : string -> unit = <fun>
```

Counting /etc/passwd gives:

```
#wc "/etc/passwd";;
16624 characters, 203 lines.
- : unit = ()
```

10.1, p. 76

Let us recall the definitions of the type token and of the lexical analyzer:

```
#type token =
#  PLUS | MINUS | TIMES | DIV | LPAR | RPAR
#| INT of int;;
Type token defined.

#(* Spaces *)
#let rec spaces = function
#  [< '` `|`\t`|`\n`; spaces _ >] -> ()
#| [< >] -> ();;
spaces : char stream -> unit = <fun>

#(* Integers *)
#let int_of_digit = function
#  `0`..`9` as c -> (int_of_char c) - (int_of_char `0`)
#| _ -> raise (Failure "not a digit");;
int_of_digit : char -> int = <fun>

#let rec integer n = function
#  [< ' `0`..`9` as c; (integer (10*n + int_of_digit c)) r >] -> r
#| [< >] -> n;;
integer : int -> char stream -> int = <fun>

#(* The lexical analyzer *)
#let rec lexer s = match s with
```

```
#  [< ''('; spaces _ >] -> [< 'LPAR; lexer s >]
#| [< '')'; spaces _ >] -> [< 'RPAR; lexer s >]
#| [< ''+'; spaces _ >] -> [< 'PLUS; lexer s >]
#| [< ''-'; spaces _ >] -> [< 'MINUS; lexer s >]
#| [< ''*'; spaces _ >] -> [< 'TIMES; lexer s >]
#| [< ''/'; spaces _ >] -> [< 'DIV; lexer s >]
#| [< ''0'..'9' as c; (integer (int_of_digit c)) n; spaces _ >]
#                                -> [< 'INT n; lexer s >];;
lexer : char stream -> token stream = <fun>
```

The parser has the same shape as the grammar:

```
#let rec expr = function
#  [< 'INT n >] -> n
#| [< 'PLUS; expr e1; expr e2 >] -> e1+e2
#| [< 'MINUS; expr e1; expr e2 >] -> e1-e2
#| [< 'TIMES; expr e1; expr e2 >] -> e1*e2
#| [< 'DIV; expr e1; expr e2 >] -> e1/e2;;
expr : token stream -> int = <fun>

#expr (lexer (stream_of_string "1"));;
- : int = 1

#expr (lexer (stream_of_string "+ 1 * 2 4"));;
- : int = 9
```

10.2, p. 77

The only new function that we need is a function taking as argument a character stream, and returning the first identifier of that stream. It could be written as:

```
#let ident_buf = make_string 8 ` `;;
ident_buf : string = "        "

#let rec ident len = function
#  [< ' `a`..`z`|`A`..`Z` as c;
#     (if len >= 8 then ident len
#      else begin
#            set_nth_char ident_buf len c;
#            ident (succ len)
#           end) s >] -> s
#| [< >] -> sub_string ident_buf 0 len;;
ident : int -> char stream -> string = <fun>
```

The lexical analyzer will first try to recognize an alphabetic character c, then put c at position 0 of `ident_buf`, and call `ident 1` on the rest of the character stream. Alphabetic characters encountered will be stored in the string buffer `ident_buf`, up to the 8th. Further alphabetic characters will be skipped. Finally, a substring of the buffer will be returned as result.

```
#let s = stream_of_string "toto 1";;
s : char stream = <abstr>

#ident 0 s;;
- : string = "toto"

#(* Let us see what remains in the stream *)
#match s with [< 'c >] -> c;;
- : char = ` `

#let s = stream_of_string "LongIdentifier ";;
s : char stream = <abstr>

#ident 0 s;;
- : string = "LongIden"

#match s with [< 'c >] -> c;;
- : char = ` `
```

The definitions of the new `token` type and of the lexical analyzer is trivial, and we shall omit them. A slightly more complex lexical analyzer recognizing identifiers (lowercase only) is given in section 12.2.1 in this part.

11.1, p. 83

```
(* main.ml *)
let chars = counter__new 0;;
let lines = counter__new 0;;

let count_file filename =
  let in_chan = open_in filename in
    try
      while true do
        let c = input_char in_chan in
          counter__incr chars;
          if c = `\n` then counter__incr lines
      done
    with End_of_file ->
      close_in in_chan
;;

for i = 1 to vect_length sys__command_line - 1 do
  count_file sys__command_line.(i)
done;
print_int (counter__read chars);
print_string " characters, ";
print_int (counter__read lines);
print_string " lines.\n";
exit 0;;
```

Chapter 18

Conclusions and further reading

We have not been exhaustive in the description of the Caml Light features. We only introduced general concepts in functional programming, and we have insisted on the features used in the prototyping of ASL: a tiny model of Caml Light typing and semantics.

The reference manual [21] provides an exhaustive description of the Caml Light language, its libraries, commands and extensions.

Those who read French are referred to [38], a progressive, but thorough introduction to programming in Caml Light, with many interesting examples, and to [19], the French edition of the Caml Light reference manual.

Description about "Caml Strong" and useful information about programming in Caml can be found in [9] and [37].

An introduction to lambda-calculus and type systems can be found in [17], [12] and [4].

The description of the implementation of call-by-value functional programming languages can be found in [20].

The implementation of lazy functional languages is described in [29] (translated in French as [30]). An introduction to programming in lazy functional languages can be found in [5].

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