PREFMAP-3 User's Guide

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1986

The PREFMAP-3 project was started when the two first authors were staying at Bell Labs in 1982. The authors would like to thank Joseph Kruskal for his comments and advice regarding the output of the computer program. Thanks are also due to Suzanne Winsberg and Sandra Pruzansky for their comments on an earlier version of the manuscript.

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Introduction

Preference mapping (PREFMAP) is a basic statistical technique for the behavioral and social sciences, particularly psychology and marketing, and has wide potential for application in other areas as well (e.g., speech research, linguistics or psychoacoustics). Briefly speaking, its purpose is to relate preference information on a number of objects to a pre-existing spatial configuration of points, under the assumption of a simple class of models. Here "preference" is used as a generic name of any type of observations that indicate a conditional dominance relation among the objects; thus "properties", "attributes", "Q-sorts", or – in still another language – "variables" defined on "cases" can also be modeled with preference mapping. What makes PREFMAP clearly distinct from a straightforward correlation approach is the possibility to go beyond monotonically increasing preference (or response-) functions, and to study various forms of *single-peakedness.* This concept has a long and complicated history in the behavioral sciences, some of which is touched upon in Coombs and Avrunin (1977), and Heiser (1981). The first regular description of the PREFMAP methodology as a hierarchy of models and techniques was given by Carroll (1972). Further useful references are Carroll (1980), Heiser and De Leeuw (1981), and Coxon (1982).

This manual provides a comprehensive account of the PREFMAP models and techniques in connection with the PREFMAP-3 program. PREFMAP-3 is the successor to PREFMAP and PREFMAP-2 (Chang and Carroll, 1972), preserving most of their features, but completely redesigned and reprogrammed in order to obtain a greater flexibility, portability and capacity. An outline of the differences is included in Appendix A, which also gives technical details and some instructions for getting the program running properly. Nowhere, is familiarity with the previous versions assumed.

The manual is organized into two parts. Part I contains a general introduction to the PREFMAP hierarchy of models and the associated hierarchy of regression equations for fitting the models to data. It also contains a section on preliminary transformations of the spatial configuration to improve fit, and a short introduction to the way in which nonmetric analyses are performed. Part II provides practical guidance in the use of the PREFMAP-3 program. The so-called standard input stream is described, and the various mechanisms available to control the behavior of the program. There is also an explanation of the layout of the output, and of the trouble reports and program-specific warnings that may be issued during execution. Part II concludes with some applications, merely giving a glimpse of the possibilities. Finally, Appendix B is a detailed statement of the input records and their organization, attached there for ease of references.

We welcome feedback from the users of PREFMAP-3, such as applications of special interest, and novel but useful ways of using the program. We also welcome information about any errors which may remains in the program, especially if accompanied by sufficient information to permit tracking them down.

Part I

PREFMAP models and techniques

Chapter 1

Basic data and objectives

The basic data for a PREFMAP analysis consists of two parts. The first art is called the *external data matrix*, and can be depicted as follows.

$$\begin{array}{c|ccccc|}
 & 1 & \dots & j & \dots & m \\
\hline
1 & & & & & \\
\vdots & & & & & \\
i & & & \delta_{ij} & & \\
\vdots & & & & & \\
n & & & & & \\
\hline
\end{array}$$

Figure 1.1: The external data matrix

The external data matrix contains entries δ_{ij} that are interpreted as measures of the *dissimilarity* between row-object i and column-object j. In a psychological context the row-objects are often called subjects, and the column objects stimuli. The dissimilarities could be derived from preference judgments, in which case they could be interpreted as measures of relative distance from the subjects' "ideal stimulus points" in the space (a different "ideal point" being assumed for each subject, which may, in the case of a model called the "vector model", be infinitely distant from the actual stimuli, so that only a direction – indicated by a subject "vector" – is defined). The analysis of this type of data, representing both the row- and the column-objects in a p-dimensional space such that the Euclidean distances d_{ij} between row-point i and column-point j resemble as closely as possible the dissimilarities δ_{ij}, has become known as *Multidimensional Unfolding*. The type of analysis that is performed by the PREFMAP-3 program is sometimes called *External Unfolding*. Contrary to Internal Unfolding, where both the row-points and the column-points must be solved for, External Unfolding finds a representation for each row-object conditionally upon *given* points for the column- objects.

This situation defines the second part of the basic input as the target configuration (see Figure 1.2). The rows of the target configuration must correspond to the columns of the data matrix. The entries y_{js} are the coordinate values of the m points in p-dimensional space. These values may be obtained from any multidimensional data analysis technique, or be derived from some predefined

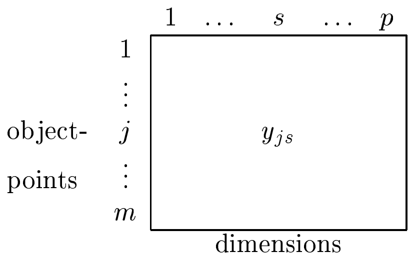

Figure 1.2: The external data matrix

structure. For historical reasons, the technique that relates the external data to the given configuration is called PREFerence Mapping. One should realize, however, that the entries of the data matrix can consist of any score, rating or ranking by the rows (subjects, scales, variables, properties) to the columns (stimuli, objects, cases). From now on, we shall adhere to the convention to denote the row-entities as *individuals* and the column-entities as *objects*, and to assume that the scoring direction is: "small value ↔ high preference" and "large values ↔ low preference" (hence that δ_{ij} are sometimes mnemonically called dispreferences). If the original data are preferences (or other *similarity* measures) the values should be reversed in order to make them dispreferences (or *dissimilarities*) say by subtracting them all from some large number, or simply reversing their signs. (At present, this step must be taken *before* input to PREFMAP-3.)

The individuals can be represented in the target configuration for the objects in two different ways: either as a *vector* (which can be thought of as indicating the direction of an infinitely distant ideal point, as discussed earlier), or as an *ideal point*. In the former representation, preference increases monotonically along one single direction in space (indicated by the vector). In the simplest case of the latter type of representation there is one single point of maximum preference (called the ideal point), and preference decreases along all directions in space as a function of the distance from the ideal point. Thus in both cases the preference function is single-peaked, provided we imagine the vector representation as a "ridge of peaks" at infinity. As we shall see shortly there are variations on this basic scheme that allow us to study single-dipped functions, when there exists a point of minimum preference, or mixtures of single-peakedness single-dippedness as well. The whole range of models forms a nested sequence, or a hierarchy, and will be discussed in chapter 2 in order of complexity.

For each member of the hierarchy, a specific set of regression equations can be used to obtain least squares estimates of the model parameters. A detailed derivation is given in chapter 3. For one member of the hierarchy - the simple Unfolding or ideal point model - it can be useful to transform the target configuration before the mapping process starts; these preliminary transformations are fully explained in chapter 4. Regardless of the chosen model, the analysis can be done metrically or nonmetrically (chapter 5). In the former case the relation between the values predicted by the preference function and the data is assumed to be linear, in the latter case merely the order of the data values is taken into account. This option again enlarges the class of possible preference functions a great deal (in anticipation we already freely used expressions like "monotonically increasing preference", it being understood that in a metric analysis monotonicity is constrained to be linear).

Due to the conditional nature of the preference mapping (i.e., a fixed target configuration), each individual can be modeled independently from the others. There are definite advantages to considering groups of individuals simultaneously (therefore Figure 1.1 is a table, not a single row of dissimilarities). But this circumstance does not necessarily urge us to choose the same model for each individual.

Chapter 2

The PREFMAP hierarchy of models

2.1 The vector model

The vector model is the simplest in the hierarchy. The individual is represented in the p-dimensional configuration by a vector. The direction of the vector is indicative of increasing preference. More in particular, the dissimilarity values are approximated by the perpendicular projection of the object points onto the individual vector (where, in PREFMAP-3, the signs are adjusted to let a high projection value correspond with a low dissimilarity value, i.e., by convention, a high preference).

Although the representation of a row of the data matrix is very similar to the results for, say a variable in Principal Components analysis, we cannot conclude anything from a correlation point of view without restraint. Only when the target configuration is orthonormal, i.e. the coordinate values have sum of squares one and are uncorrelated, the angles between vectors and the axes of the configuration can be interpreted as correlations. When this condition is not satisfied the vector must simply be interpreted as the best *direction* of overall increasing preference.

2.2 The Unfolding model

This model is also called the ideal point model, or the simple Euclidean model. The individual is represented as a point, located at a position of an imaginary object point that would receive maximum preference value (called the ideal point). The dissimilarities are approximated by the squared Euclidean distances between the ideal point and the object points. Here it is quite natural that a small dissimilarity is represented by a small distance: the ideal point will be close to the objects the individual prefers most. This explains the dissimilarity/dispreference scoring convention (the Unfolding model is, in many ways, most central in preference mapping). The name Unfolding originates from the following metaphor. If we imagine - in one or two dimensions - the object points as spots on a napkin, next pick it up at the ideal point position and *fold* it, then the spots will appear on the the folded napkin in order of preference. The data analysis takes us in the reverse direction, hence the name Unfolding.

2.3 The weighted Unfolding model

The weighted Unfolding or weighted ideal point model also depicts the individual as a point, but now the model allows any individual to (re)weight the dimensions in his/her own way. The weights can be thought of as importances of dimensions for a specific individual. Accordingly, the dissimilarities are approximated by weighted squared distances, with a distinct pattern of weights for every individual.

In terms of a preference function, more in particular a preference surface, the weighted Unfolding model still implies a single peak at the location of the ideal point, but now preference decreases more rapidly in the direction of a more heavily weighted axis (and less rapidly along a less heavily weighted one). The contours of equal preference are ellipse (or ellipsoid) centered at the ideal point, with axes parallel to the coordinate axes, and lengths of axes inversely related to the weights. The simple Unfolding model is of course the special case of equal weights, so that the ellipses (ellipsoids) become circles (spheres) centered at the ideal point.

2.4 The general Unfolding model

Distances between an ideal point and the object points are invariant under orthogonal rotation of the complete configuration. They do become different, however, when we first allow an optimal rotation and then apply weights to this new set of reference axes (and, if desired, rotate back). This idea constitutes the final ideal point model, sometimes called the general Euclidean model. An individual is allowed to rotate the target configuration before weighting the new (rotated) axes. Thus a group of individuals all modeled this way can show differences in three respects: ideal point location, axis orientation, and importance of the reoriented axes.

It will be clear that this generalization implies that the contours of equal preference still are ellipses (in more than two dimensions: ellipsoids) centered at the ideal point. For the general Unfolding model is equivalent to the weighted Unfolding model *after* the individual reorientation has been effected. Then, keeping the position of the target configurations constant, ellipses of different individuals will all have different orientations.

2.5 Weights and their signs

The parameters of the vector model are p numbers fixing the direction of the individual vector with respect to the coordinate axes of the target configuration. If normalized (sum of squares equal to 1), these p numbers are called direction cosines. In another context, they are sometimes called "weights", but we have to be careful in PREFMAP not to confuse them with the weights of the weighted Unfolding model. It is not true, for instance, that the vector model is a special case of the weighted Unfolding model in which the ideal point is located in the origin. The unfolding model "weights" apply to coordinate-wise squared differences (between the ideal point and an object point); the vector model "weights" weight contributions to a linear sum of coordinate values (for each object point). If it is not in terms of "weights", in what sense, then, is the vector model a special case of Unfolding model? When we move the ideal point outwards along a fixed direction, say v, then the contours of equal preference become circles (or spheres in higher dimensions) with larger and larger radii. If moved far enough, the circle (sphere) segments in the neighborhood of the object points are approximately straight lines (planes or hyperplanes) perpendicular to v, and this makes them indistinguishable from lines (hyperplanes) of equal preference for a vector model with vector v.

Actually, the three ideal point models all accommodate weights. While in the weighted and the general Unfolding model a *weight pattern* is obtained, in the simple Unfolding model the *magnitude* of the weights is equal to *one* for each dimension. Now, there is nothing in the method that fits the models to the external data that ensures the weights to be positive (see chapter 3). In the case of the simple ideal point model the weights for each dimension can become minus one. We cannot interpret the individual point any longer as an ideal point, but have to appreciate it as an *anti-ideal point*: the (squared) distance between an individual point and an object point will be small when the dissimilarity is large. Preferences are represented in a way that shows how much an individual *dislikes* an object. For the more complicated ideal point models, the weight pattern can also turn out to be more complicated. The same anti-ideal point interpretation holds when the weights for each dimension are negative. When they are positive for some dimensions and negative for others, the individual point is called a *saddle point*. Along the dimensions with positive weights distance decreases when preference increases, and for the dimensions with negative weights it is just the other way around.

Finally a situation must be anticipated that complicates the interpretation of the individual point for the simple ideal point. This might come out when a preliminary transformation of the target configuration is performed (see chapter 4, for a description and a mathematical account). When a preliminary transformation is allowed for, we obtain equal weights with a *sign pattern* for the ideal point model instead of a single weight with an equal sign for each dimension. In that case we have to interpret the individual point in the same way as we would do for the weighted and the general model; i.e., we have to decide by inspecting the signs of the weights whether the individual point is a saddle point instead of simply an ideal or an anti-ideal point. In all cases with mixed sign combinations the preference surface is no longer single-peaked or single-dipped; this may occasionally give problems in interpretation, but there seem to be no compelling general reasons for excluding these possibilities from the hierarchy altogether. Another target configuration might be tried, which is often a good idea anyhow.

Chapter 3

The PREFMAP hierarchy of regression equations

Both the vector model and the Unfolding models can be expressed as a set of equations that are linear in the unknowns. These equations can be approximately solved by ordinary least squares regression if we perform appropriate changes of variables and reparametrizations. Carroll (1972) is the major reference here, see also Carroll (1980) and Heiser and De Leeuw (1981). It will be shown in the next paragraphs what operations have to be performed, starting again with the vector model. In all models the data are assumed to be at the interval level, so that the obtained model values must remain invariant under linear transformations of the data (two parameters, a_i and b_i, are introduced to take care of this). The symbol $\simeq$ is used to indicate least squares approximation, and R_i denotes the multiple correlation coefficient. In all formulas the index i is retained to refer to individual i, although strictly speaking it is superfluous (different individuals can be modeled differently and independently). In particular, the introduction of some model parameter, e.g. w_{is}, does not imply that it has to be there for all $i = 1, \ldots, n$. Whenever two parameters are confounded, the identification conditions selected in PREFMAP-3 are explicitly stated.

3.1 Equations for the vector model

In the vector model dissimilarities δ_{ij} are assumed to be approximated as

$$\delta_{ij} \simeq a_i \sum_{s=1}^{p} x_{is} y_{js} + b_i. \tag{3.1}$$

Here a_i is the slope parameter, b_i is the intercept term, x_{is} is the vector coordinate in dimension s, and y_{js} is the target point coordinate; δ_{ij} and y_{js} are known; a_i, b_i and x_{is} are unknown. Now we re-express the system using the following convention: the index q of the system, and s or other indices of the old system are coupled in order of enumeration.

Change of variables	Reparametrization	Index ranges and definition
$z_{j0}=1$	$g_{i0}=b_i$	$q=0$
$z_{jq}=y_{js}$	$g_{iq}=a_i x_{is}$	$q=s$ $(s, q = 1, \ldots, p)$

Substitution into equation (3.1) gives us the transformed model

$$\delta_{ij} \simeq g_{i0} z_{j0} + \sum_{q=1}^{p} g_{iq} z_{jq} = \sum_{q=0}^{p} g_{iq} z_{jq}. \tag{3.2}$$

Thus we end up with a set of m nonhomogeneous linear equations in $p+1$ unknowns. These can be approximately solved (in a least squares sense) by multiple regression techniques. The predictor set contains a vector of ones plus p vectors of target coordinates. The dissimilarities δ_{ij} will function as the criterion values. Once the regression weights in equation (3.2) are determined, we find values for the parameters of the original model by applying

$$\begin{aligned} a_i &= \left(\sum_{q=1}^{p} g_{iq}^2 / R_i^2 \max_j \sum_{s=1}^{p} y_{js}^2 \right)^{1/2}, \\ x_{is} &= g_{iq} / a_i, \\ b_i &= g_{i0}. \end{aligned}$$

In fact the normalization of the vector coordinates is free to be chosen. In PREFPAM-3 it has been decided to normalize them in such a way that the length of the vector is proportional to the fit R_i. The overall size is determined by the target point that has largest distance from the origin of the configuration. In this way the length of the vectors will harmonize with the size of the target configuration, while still displaying the relative fit.

3.2 Equations for the Unfolding model

Here the dissimilarities are assumed to be approximated as

$$\delta_{ij} \simeq a_i d_{ij}^2 + b_i, \tag{3.3}$$

with d_{ij}^2 the squared Euclidean distance between ideal point i and target point j. Rewriting (3.3) with coordinates values x_{is} and y_{js} gives us

$$\delta_{ij} \simeq a_i \left\{ \sum_{s=1}^{p} (x_{is} - y_{js})^2 \right\} + b_i = a_i \left\{ \sum_{s=1}^{p} x_{is}^2 - 2 \sum_{s=1}^{p} x_{is} y_{js} + \sum_{s=1}^{p} y_{js}^2 \right\} + b_i. \tag{3.4}$$

Change of variables	Reparametrization	Index ranges and definition
$z_{j0}=1$	$g_{i0}=a_i \sum_{s=1}^{p} x_{is}^2 + b_i$	$q=0$
$z_{jq}=y_{js}$	$g_{iq}=-2a_i x_{is}$	$q=s$ $(s, q = 1, \ldots, p)$
$z_{j(p+1)}=\sum_{s=1}^{p} y_{js}^2$	$g_{i(p+1)}=a_i$	$q=p+1$

Transformed model

$$\delta_{ij} \simeq g_{i0} z_{j0} + \sum_{q=1}^{p} g_{iq} z_{jq} + g_{i(p+1)} z_{j(p+1)} = \sum_{q=0}^{p+1} g_{iq} z_{jq}. \tag{3.5}$$

Now the transformed model is a set of m linear equations in $p+2$ unknowns. The predictor set contains, in addition to the vector of ones and the target coordinates, the sum of squares of the target coordinates $(z_{j(p+1)})$. The parameters of the original model are obtained as

$$\begin{aligned} a_i &= |g_{i(p+1)}|, \\ x_{is} &= -\frac{1}{2} g_{iq}/g_{i(p+1)}, \\ b_i &= g_{i0} - a_i \sum_{s=1}^{p} x_{is}^2. \end{aligned}$$

The slope is always identified as a nonnegative quantity; the sign of the regression weight $g_{i(p+1)}$ determines whether we deal with an ideal point or an anti-ideal point. When this regression weight approaches zero, the ideal point will move to infinity.

3.3 Equations for the weighted Unfolding model

This model is equivalent to the Unfolding model in equation (3.3), except that the distances are defined by

$$d_{ij}^2 = \sum_{s=1}^{p} w_{is}(x_{is} - y_{js})^2 \tag{3.6}$$

and thus (3.4) can be written for the weighted model as

$$\delta_{ij} \simeq a_i \left\{ \sum_{s=1}^{p} w_{is} x_{is}^2 - 2 \sum_{s=1}^{p} w_{is} x_{is} y_{js} + \sum_{s=1}^{p} w_{is} y_{js}^2 \right\} + b_i \tag{3.7}$$

giving for the new system

Change of variables	Reparametrization	Index ranges and definition
$z_{j0}=1$	$g_{i0}=a_i \sum_{s=1}^{p} w_{is} x_{is}^2 + b_i$	$q=0$
$z_{jq}=y_{js}$	$g_{iq}=-2a_i w_{is} x_{is}$	$q=s$ $(s, q = 1, \ldots, p)$
$z_{jq}=y_{js}^2$	$g_{iq}=a_i w_{is}$	$q=p+s$ $(s = 1, \ldots, p;$ $q = p+1, \ldots, 2p)$

Transformed model

$$\delta_{ij} \simeq g_{i0} z_{j0} + \sum_{q=1}^{2p} g_{iq} z_{jq} = \sum_{q=0}^{2p} g_{iq} z_{jq}. \tag{3.8}$$

Under the weighted Unfolding model we thus have to determine $2p+1$ regression weights. For that purpose the predictor matrix must contain, apart from the vector of ones and p vectors with the target coordinates, another p vectors with the squares of the target coordinates. The solution for the original parameters is:

$$\begin{aligned} a_i &= \left(\tfrac{1}{p} \textstyle\sum_{q=p+1}^{2p} g_{iq}^2 \right)^{1/2}, & & \\ w_{is} &= g_{iq}/a_i, & & q = (p+1), \ldots, 2p, \\ x_{is} &= -\tfrac{1}{2} g_{iq}/g_{i(q+p)}, & & q = 1, \ldots, p, \\ b_i &= g_{i0} - a_i \textstyle\sum_{s=1}^{p} w_{is} x_{is}^2. & & \end{aligned}$$

By choosing this identification for the a_i, the weights w_{is} are normalized so that their sum of squares equals the number of dimensions p. The sign pattern of the w_{is}'s indicates whether we deal with an ideal point (all signs positive), an anti-ideal point (all signs negative), or a saddle point (some signs positive, the others negative).

3.4 Equations for the general Unfolding model

The general Unfolding model is essentially the same as the weighted ideal point model, apart from the fact that both the ideal point and the target points are jointly reoriented by an orthogonal rotation matrix $\mathbf{T}_i$. When we define $\mathbf{X}^* = \mathbf{X}\mathbf{T}_i$ and $\mathbf{Y}^* = \mathbf{Y}\mathbf{T}_i$ the distances are given by

$$d_{ij}^2 = \sum_{s=1}^{p} w_{is}(x_{is}^* - y_{js}^*)^2. \tag{3.9}$$

Defining the transformation matrix $\mathbf{R}_i = \mathbf{T}_i\mathbf{W}_i\mathbf{T}_i'$, with $\mathbf{W}_i$ a diagonal matrix containing the individual dimension weights, the regression equations for the general Unfolding model are

$$\begin{aligned}\delta_{ij} \simeq a_i \Bigg\{ & \sum_{s=1}^{p}\sum_{u=1}^{p} x_{is} r_{su}^i x_{iu} \\ & -2\sum_{s=1}^{p}\sum_{u=1}^{p} x_{is} r_{su}^i y_{ju} + \sum_{s=1}^{p}\sum_{u=1}^{p} y_{js} r_{su}^i y_{ju} \Bigg\} + b_i. \end{aligned} \tag{3.10}$$

(The notational convention used here is that the general entry in a matrix denoted by a bold capital letter with *sub*script i will be indicated by the same letter, doubly subscripted and in small case with a *super*script i; e.g. r_{su}^i is the (s,u) element of $\mathbf{R}_i$.)

Change of variables	Reparametrization	Index ranges and definition
$z_{j0}=1$	$g_{i0}=a_i \sum_{s=1}^{p}\sum_{u=1}^{p} x_{is} r_{su}^i x_{iu} + b_i$	$q=0$
$z_{jq}=y_{js}$	$g_{iq}=-2a_i \sum_{u=1}^{p} x_{iu} r_{su}^i$	$q=s$ $(s, q = 1, \ldots, p)$
$z_{jq}=y_{js}^2$	$g_{iq}=a_i r_{ss}^i$	$q=p+s$ $(s = 1, \ldots, p;$ $q = p+1, \ldots, 2p)$
$z_{jq}=y_{js}y_{ju}$	$g_{iq}=2a_i r_{su}^i (= 2a_i r_{us}^i)$	$q=2p+s(s-1)/2+u-1$ $(s < u = 1, \ldots, p;$ $q = 2p+1, \ldots, p(p+3)/2)$

Transformed model

$$\begin{aligned}\delta_{ij} &\simeq g_{i0}z_{j0} + \sum_{q=1}^{p} g_{iq}z_{jq} + \sum_{q=p+1}^{2p} g_{iq}z_{jq} + \sum_{q=2p+1}^{p(p+3)/2} g_{iq}z_{jq} \\ &\simeq \sum_{q=0}^{p(p+3)/2} g_{iq}z_{jq}. \end{aligned} \tag{3.11}$$

For the general model we can estimate the values of parameters by determining $p(p+3)/2$ regression weights. In addition to the predictor matrix for the weighted Unfolding model, we have $p(p-1)/2$ vectors of cross product terms of the target coordinates. The solution in terms of the original parameters is

$$
\begin{array}{lll}
a_i = \left(\frac{1}{p}\sum_{q=p+1}^{2p} g_{iq}^2\right)^{1/2} & & \\
r_{ss}^i = g_{iq}/a_i, & q=p+1,\ldots,2p & (s=q) \\
r_{su}^i (=r_{us}^i) = \frac{1}{2} g_{iq}/a_i, & q=2p+1,\ldots,p(p+3)/2 & (s = \text{integer} \\
 & & s' \ni s'(s'-1)/2 < q' \text{ and} \\
 & & s'(s'+1)/2 \geq q' \text{ while} \\
 & & u = q'-s, \text{ where} \\
 & & q' = q-2p) \\
b_i = g_{i0} - a_i \sum_{s=1}^{p}\sum_{u=1}^{p} x_{is} r_{su}^i x_{iu} & & .
\end{array}
$$

The transformation matrix $\mathbf{R}_i$ can be decomposed to obtain the weighted matrix $\mathbf{W}_i$ and the rotation matrix $\mathbf{T}_i$, by solving for the eigenvalues and eigenvectors of $\mathbf{R}_i = \mathbf{T}_i \mathbf{W}_i \mathbf{T}_i'$. Next the ideal point coordinates can be obtained as

$$x_{is} = -\frac{1}{2a_i} \sum_{q=1}^{p} g_{iq} \sum_{u=1}^{p} t_{qu}^i t_{su}^i / w_{uu}^i, \; s = 1, \ldots, p.$$

This settles the calculations for the general Unfolding model.

3.5 Model testing: F-statistics

Summarizing the results of the previous sections we obtain the following table of variables that are involved in the parameter estimation of the various models (Table 3.1).

Table 3.1: The set of predictor variables.

	vector of ones	Coordinates	SSQ's	SQ's	Cross-products	Total number of parameters
Model	1	y_{js}	$\sum_{s=1}^{p} y_{js}^2$	y_{js}^2	$y_{js}y_{ju}$	k
Vector	x	x				$p+1$
Unfolding	x	x	x			$p+2$
Weighted	x	x		x		$2p+1$
General	x	x		x	x	$p(p+3)/2$

Note that for any model the total number of parameters, k, must be smaller than or equal to the number, m, of target points, y_j $(j = 1, \ldots, m)$. If this is not the case, and the model is nevertheless asked for, the PREFMAP-3 program will print a warning message and will not apply the model.

Since for the models we use regression equations that are linear in the parameters, we could use the squared multiple correlation coefficient (the squared fit) to compute F-ratios in order to test null hypotheses. Some precautions are warranted, however, because the observations might not be independent (e.g., preference judgments obtained from one individual). Lack of independence will generally inflate the significance levels; therefore one should be very conservative about them. Moreover, significance tests are only one criterion among many others to decide which model fits the best data.

The F-statistics are computed by PREFMAP-3 are based on the squared *metric* fit and the number of fitted parameters involved (k, as defined in Table 3.1). The total number of data values is m. For testing the null hypothesis $\rho_k = 0$ against the alternative hypothesis $\rho_k > 0$, under the assumption of normally distributed errors, the following F-ratio and accompanying degrees of freedom apply:

$$\frac{R_k^2/(k-1)}{(1-R_k^2)/(m-k)}, \text{ with } k-1 \text{ and } m-k \ df.$$

Here R_k is the empirical estimate of ρ_k, the multiple correlation with k predictors. When $m = k$ we will obtain perfect fit and the ratio is not defined. For testing the hypothesis $\rho_a = \rho_b$, where model a is the more complex model, the following ratio applies:

$$\frac{(R_a^2 - R_b^2/(k_a - k_b)}{(1-R_a^2)/(m-k_a)}, \text{ with } k_a - k_b \text{ and } m - k_a \ df,$$

which can also be compared with the tabulated values for the F-distribution.

Chapter 4

Preliminary transformations of the target configuration

4.1 Room for improvement under the simple Unfolding model

In some cases it is suitable to allow the target configuration to be transformed before a group of individuals is fitted into it. There are two possible linear transformations, which in spirit resemble the weighted and the general Unfolding model: the original axes of the target configuration can be differentially "stretched", or a new set of reference axes can be obtained by weighting after orthogonal rotation. Either transformation will apply to all individuals, and is optimal in the sense that the *proportion of total variance accounted for* by the simple Unfolding model will be maximal. The proportion of total variance accounted for equals the average squared fit across individuals, for the *metric* case.

There are a number of situations where this type of preliminary transformation seems suitable. When, e.g., the target configuration is the group stimulus space from an INDSCAL individual differences scaling analysis (cf. Carroll and Chang, 1970), the object point coordinates are normalized for each axis. By performing a preliminary weighting of the axes the configuration might become more meaningful. Note that the full transformation, i.e. rotation and weighting, would not be wise in this case since the orientation of axes obtained by an INDSCAL analysis is uniquely related to the individual weights from that analysis. Another application might be the case where we want to use a target configuration that has been constructed from a set of hypothetical variables. Since we would not be sure of their proper orientation and their relative importance, it might be enlightening to allow a preliminary rotation and stretching (reweighting) of the axes.

It is important to bear in mind that both options for the transformation are especially designed for the simple ideal point model. The weighted and the general ideal point model already provide optimal weights by themselves, while the vector model will absorb weights in the vector coordinates. Therefore we will obtain the same fit when applying, e.g., the weighted Unfolding model directly compared to performing optimal weighting beforehand and next fitting the weighted model. The full transformation will affect both the weighted and the simple ideal point model, but it is optimal only for the latter. The vector model and the general model are not affected by it, at least as far as the fit is concerned.

When a preliminary transformation is called for, the optimal configuration is always determined across all rows, no matter what models are fitted in the external analysis. This also implies that a special feature of external analyses is lost when applying a preliminary transformation. When we do not ask for a transformation, the results across individuals are invariant under different selections of subgroups of individuals in the analysis (since individual results are obtained by separate multiple regressions). When we do ask for a transformation, this is no longer true, because the optimal configuration is solved for *given the selection of individual data* in the analysis. Thus although the actual preference mapping consists of separate regressions, the preliminary transformation is determined "jointly" across individuals.

In the next two sections we show how the transformed target configuration is obtained. The solution for the full transformation can also be found in Carroll (1980), the explicit solution for the weighted case is new.

4.2 The preliminary full transformation

The problem that has to be solved can be written as

$$\delta_{ij} \simeq a_i d_{ij}^2 + b_i^*, \tag{4.1}$$

with

$$d_{ij}^2 = (\mathbf{x}_i - \mathbf{T}\mathbf{y}_j)'(\mathbf{x}_i - \mathbf{T}\mathbf{y}_j), \tag{4.2}$$

where $\mathbf{T}$ is a general linear transformation matrix defined as $\mathbf{T} = \mathbf{WR}$, with $\mathbf{R}$ an orthogonal rotation matrix and $\mathbf{W}$ a diagonal weights matrix. We use the notational convention here to denote the i'th row of $\mathbf{X}$ and the j'th row of $\mathbf{Y}$ by column vectors $\mathbf{x}_i$ and $\mathbf{y}_j$, resp. Substituting (4.2) into (4.1) gives us:

$$\delta_{ij} \simeq a_i \mathbf{x}_i'\mathbf{x}_i - 2a_i\mathbf{x}_i'\mathbf{T}\mathbf{y}_j + a_i\mathbf{y}_j'\mathbf{T}'\mathbf{T}\mathbf{y}_j + b_i^*. \tag{4.3}$$

When we define $\mathbf{C} = \mathbf{T}'\mathbf{T}$, and in addition the matrices $\mathbf{V}$, $\mathbf{B}$, $\mathbf{U}$, and the vector $\mathbf{w}$ (using the same coupling of indices q and s as before):

$$\begin{array}{lll}
v_{qj} = y_{js} & b_{iq} = -2a_i\mathbf{x}_i'\mathbf{T} & q = s \ (s = 1, \ldots, p) \\
v_{(p+1)j} = 1 & b_{i(p+1)} = a_i\mathbf{x}_i'\mathbf{x}_i + b_i^* & (q = p+1) \\
u_{qj} = y_{js}^2 & w_q = c_{ss} & q = s \ (s = 1, \ldots, p) \\
u_{qj} = 2y_{js}y_{jt} & w_q = c_{st} & \left\{ \begin{array}{l} q = p - 1 + s(s-1)/2 + t \\ s < t = 1, \ldots, p; \\ q = p+1, \ldots, p(p+1)/2 \end{array} \right.
\end{array}$$

then equation (4.3) can be written as:

$$\delta_{ij} \simeq \sum_{q=1}^{p+1} b_{iq}v_{qj} + a_i \sum_{q=1}^{p(p+1)/2} w_q u_{qj} \tag{4.4}$$

or, in matrix notation

$$\mathbf{\Delta} \simeq \mathbf{BV} + \mathbf{aw'U}. \tag{4.5}$$

Note that $\mathbf{\Delta}$, $\mathbf{U}$, and $\mathbf{V}$ are given, while $\mathbf{a}$, $\mathbf{w}$, and $\mathbf{B}$ are to be solved for so as to yield a least squares fit to $\mathbf{\Delta}$. Note, also, that $\mathbf{B}$ can be *any* matrix whatever (so long as $\mathbf{T}$ is nonsingular); i.e., given any $\mathbf{B}$ we can find a set of x_i's, a_i's and b_i^*'s that would produce it. Therefore the complete squares problem

$$\min_{\mathbf{a},\mathbf{w},\mathbf{B}} SSQ(\mathbf{\Delta} - \mathbf{BV} - \mathbf{aw'U}) \tag{4.6}$$

[where SSQ is a "sum of squares" operator, i.e. $SSQ(\mathbf{A}) = \text{tr } \mathbf{AA'}$] can be simplified by splitting the sum of squares in (4.6) into two additive parts. Defining the projector matrices $\mathbf{H}$ and $\mathbf{G}$, $(\mathbf{H} + \mathbf{G} = \mathbf{I})$ as:

$$\mathbf{H} = \mathbf{I} - \mathbf{V'}(\mathbf{VV'})^{-1}\mathbf{V} = \mathbf{I} - \mathbf{G} \tag{4.7}$$

where $\mathbf{G} = \mathbf{V'}(\mathbf{VV'})^{-1}\mathbf{V}$, the additive decomposition is:

$$\begin{aligned} & SSQ(\mathbf{\Delta} - \mathbf{BV} - \mathbf{aw'U}) \\ = \ & SSQ(\mathbf{\Delta} - \mathbf{BV} - \mathbf{aw'U})(\mathbf{H} + \mathbf{G}) \\ = \ & SSQ(\mathbf{\Delta H} - \mathbf{aw'UH}) + \\ & SSQ(\mathbf{\Delta V'}(\mathbf{VV'})^{-1}\mathbf{V} - \mathbf{BV} - \mathbf{aw'UV'}(\mathbf{VV'})^{-1}\mathbf{V}). \end{aligned} \tag{4.8}$$

Now, the second sum of squares on the right-hand side of (4.7) can be made zero for any choice of $\mathbf{a}$ and $\mathbf{w}$ by defining $\mathbf{B}$ appropriately. Therefore the unknowns of primary concern, $\mathbf{w}$, can be found by minimizing the first part of the decomposition. The minimum is attained by choosing $\mathbf{w}$ as the vector maximizing the following ratio of quadratic forms:

$$\frac{\mathbf{w'}(\mathbf{UHH'\Delta'\Delta HH'U'})\mathbf{w}}{\mathbf{w'}(\mathbf{UHH'U'})\mathbf{w}}. \tag{4.9}$$

Since $\mathbf{H}$ is a projector matrix it is symmetric and idempotent; therefore the problem can be simplified into

$$\max_{\mathbf{w}} \frac{\mathbf{w'}(\mathbf{UH\Delta'\Delta HU'})\mathbf{w}}{\mathbf{w'}(\mathbf{UHU'})\mathbf{w}} \tag{4.10}$$

The maximum is found by solving a generalized eigenvalue problem. Once $\mathbf{w}$ is chosen as the eigenvector associated with the largest eigenvalue, we can apply the inverse substitution

$$\begin{array}{ll} c_{ss} = w_q & \text{for } q = 1, \ldots, p \ (s = q) \\ c_{st} (= c_{ts}) = w_q & \text{for } q = p+1, \ldots, p(p+1)/2 \\ & (s = \text{integer } s' \ni s'(s'-1)/2 < q' \\ & \ \text{while } s'(s'+1)/2 \geq q' \\ & \ \text{while } t = q' - s \text{ where } s < t = 1, \ldots, p \\ & \ \text{and } q' = q - p). \end{array}$$

Next $\mathbf{C}$ can be decomposed into $\mathbf{K\Lambda K'}$, and we solve for $\mathbf{R}$ and $\mathbf{W}$ by taking $\mathbf{R} = \mathbf{K}$ and $\mathbf{W} = \mathbf{\Lambda}^{1/2}$. Finally $\mathbf{W}$ is normalized so that its sum squares equals

p. When $\mathbf{C}$ is not positive semidefinite, some diagonal values of $\boldsymbol{\Lambda}$ are negative. In that case we take $w_q = |\lambda_q|^{1/2}$ and apply the negative sign when computing the solution for the row points $\mathbf{x}_i$, as described in section 3.2. For example: we have found the sign pattern "+ + –" from $\boldsymbol{\Lambda}$, and from the regression we obtain a negative regression weight for the quadratic term. The final weight pattern shall be "– – +", indicating a saddle point. It will be obvious from (4.4) that we at least should have $p(p+1)/2$ points $\mathbf{y}_j$ in order to be able to find a preliminary full transformation.

4.3 The preliminary weighting of axes

Under the assumption of no rotation the problem of the previous section simplifies somewhat. Equation (4.1) combined with (4.2) can now be written as

$$\delta_{ij} \simeq a_i(\mathbf{x}_i - \mathbf{W}\mathbf{y}_j)'(\mathbf{x}_i - \mathbf{W}\mathbf{y}_j) + b_i^*, \tag{4.11}$$

with $\mathbf{W}$ a diagonal weight matrix, with diagonal entries positive. In this case we use the following definitions:

$$\begin{array}{rll} v_{qj}{=}y_{js} & b_{iq}{=}-2a_i\mathbf{x}_i'\mathbf{W} & q{=}s \;(s=1,\ldots,p) \\ v_{(p+1)j}{=}1 & b_{i(p+1)}{=}a_i\mathbf{x}_i'\mathbf{x}_i + b_i^* & (q{=}p+1) \\ g_{qj}{=}y_{js}^2 & f_q{=}w_{ss}^2 & q{=}s \;(s=1,\ldots,p) \end{array}$$

which gives us

$$\delta_{ij} \simeq \sum_{q=1}^{p+1} b_{iq}v_{qj} + a_i \sum_{q=1}^{p} f_q g_{qj} \tag{4.12}$$

or, in matrix notation:

$$\boldsymbol{\Delta} \simeq \mathbf{BV} + \mathbf{af}'\mathbf{G}. \tag{4.13}$$

We end up with a problem that is completely analogous to (4.5) in the previous section. We now have to solve a (smaller) generalized eigenvalue problem to obtain the solution of

$$\max_{\mathbf{f}} \frac{\mathbf{f}'(\mathbf{GH}\boldsymbol{\Delta}'\boldsymbol{\Delta}\mathbf{HG}')\mathbf{f}}{\mathbf{f}'(\mathbf{GHG}')\mathbf{f}}. \tag{4.14}$$

From the solution for $\mathbf{f}$ we obtain the diagonal entries of $\mathbf{W}$ by defining $w_{ss} = |f_q|^{1/2}$. Taking absolute values is again necessary in the case that some elements of $\mathbf{f}$ are negative, and, as before, the sign pattern of $\mathbf{f}$ is transmitted to the point in PREFMAP-3 where the coordinate values of $\mathbf{x}_i$ are computed. The final solution for w_{ss} as given by the program again has sum of squares equal to p. To keep the projector matrix $\mathbf{H}$ well-defined, there have to be at least $p+1$ points $\mathbf{y}_j$ when preliminary weighting of axes is desired.

Chapter 5

Metric and nonmetric analyses

This terminology corresponds to the notions that are known from Multidimensional Scaling (Shepard, 1962; Kruskal, 1964; Kruskal and Shepard, 1974), and from generalizations of the Multivariate Analysis to what is called Nonlinear Multivariate Analysis (Gifi, 1981) or Optimal Scaling (cf. Young, 1981). Most closely related to the PREFMAP situation is the MONANOVA approach of Kruskal (1965).

The differences between metric and nonmetric is reflected in the way the dissimilarities are assumed to be related to the squared distances in the Unfolding models, or to the perpendicular projections in the vector model. In a metric analysis we assume this relationship to be linear, in a nonmetric analysis we allow a *monotone transformation* of the original dissimilarities. Since this transformation should be *optimal* for the squared distances (or the projections) that are fitted, we do not only have to solve for the ideal point (or vector coordinates), but for a monotone function that yields a best least squares fit as well. This chapter provides only a very brief introduction to the major ideas involved.

To keep the notation simple, we now drop nearly all reference to individual i, it being understood that everything is repeated for each $i = 1, \ldots, n$. Suppose $\{\mu_1, \ldots, \mu_m\}$ denotes the model values predicted by any member of the hierarchy of PREFMAP models, and let Ω_k denote the set of all possible model values under model k. In addition, let $\{\gamma_1, \ldots, \gamma_m\}$ denote a set of *substitute dissimilarity values*, and let Γ_i be the set of all monotonic transformations of row i of the external data matrix. A feasible substitute is a $\{\gamma_1, \ldots, \gamma_m\} \in \Gamma_i$, which is simply another notation for the requirement

$$\gamma_j \leq \gamma_l \text{ if } \delta_{ij} < \delta_{il}. \tag{5.1}$$

Thus we are satisfied with any substitute that is monotonically increasing with the data. Now the complete, *nonmetric* PREFMAP problem is to find

$$\min_{\{\mu_1,\ldots,\mu_m\}\in\Omega_k} \min_{\{\gamma_1,\ldots,\gamma_m\}\in\Gamma_i} \sum_{j=1}^{m} (\gamma_j - \mu_j)^2 \tag{5.2}$$

under the condition that the γ_j's are normalized (to avoid trivial solutions with $\gamma_j = \mu_j = 0$ for all j). This is done by using an algorithm that solves iteratively for one of the two sets of parameters at a time. Such an algorithm has become known as an ALS algorithm, the acronym for *Alternating Least Squares*. PREFMAP fits, for fixed γ (initialized as δ), the vector and distance models by solving for the parameters in a regression equation (see chapter 3, the outer minimization in (5.2)). These multiple regression calculations are then alternated with a so-called *monotone regression* of the predicted values (the inner minimization in (5.2)). The monotone regression part is performed by an algorithm as described in Kruskal (1964).

Apart from the fact that we no longer need to assume the external data to be at the interval level (the nonmetric analysis will remain invariant under all (row-wise) monotonic transformations of the dissimilarities), there can also be a consideration of simplicity. The model might be kept more simple when the data are transformed, because part of the nonlinearity (or "interaction") can be absorbed in the transformation. In terms of preference surfaces, e.g., the difference between linearly increasing and quadratically increasing slopes disappears under nonmetricity, so that the vector and the ideal point model will sooner appear to be equivalent. (That is, an ideal point sufficiently distant, but not infinitely distant, from the stimuli, will yield an order of predicted dispreferences identical to the order induced by projections on a vector in that direction.)

If the external data contain ties, i.e. equal values, then two different approaches are available to be used. The first is called the *primary approach*, and it completes (5.1) with:

$$\text{if } \delta_{ij} = \delta_{il} \text{ then } \left\{ \begin{array}{rl} \text{either} & \gamma_j \leq \gamma_l \\ \text{or} & \gamma_j > \gamma_l \end{array} \right. . \tag{5.3}$$

Thus ties in the data may become untied by the monotone regression, in either direction; the primary approach puts no additional constraints on the modeling process. In contrast, the *secondary approach* does constrain equal values to remain equal:

$$\text{if } \delta_{ij} = \delta_{il} \text{ then } \gamma_j = \gamma_l. \tag{5.4}$$

It primarily depends on the precision and reliability of the data which option is to be preferred; the secondary approach assumes more precise and reliable data, and consequently will always give a worse (or, at best, an equally bad) fit for a given model choice from the PREFMAP hierarchy.

Part II

Use of the PREFMAP-3 program

Chapter 6

Description of the input

As explained in chapter 1, there are two basic pieces of data for PREFMAP-3 to work with: an external data matrix, and a target configuration. In addition, of course, the program has to be told what exactly to do in one single run. In section 6.1 a general description of the input organization is given, while some parts of it are further explained in sections 6.2–6.6. Finally, section 6.7 provides a small working example, which will also serve in the discussion of the PREFMAP-3 output (chapter 7).

Since there any many ways to communicate with a computer, and many control languages in current use, it is useful to state explicitly what is meant by some of the words and phrases used throughout this part of the PREFMAP-3 User's Guide. The program has been written in ANSI-FORTRAN, and it obeys all rules and conventions from this standard language.

First, the input must be coded on a special type of record, called *card*, which is a record of fixed length and 80 positions long. Of course, it does not have to be an actual "card", but in some file systems care must be taken to ensure that the input has this fixed form. Secondly, cards are read by the program from a *unit*, which is an input device as defined by certain control phrases in the operating system. In what will be called the *standard input stream* it is assumed that all cards come from the same unit (PREFMAP-3 is able to read parts of the input from distinct units, cf. 6.1, 6.6). Always make sure that a card contains *blanks* on whatever positions where nothing else is intentionally specified (although often PREFMAP-3 will not react strangely to unexpected symbols). If the program expects a specification on a certain position, but encounters a blank, it will mostly perform a prechosen action called the *default*. If in the sequel it is not indicated what the default action is, this can either mean that PREFMAP-3 will not be able to respond, lacking essential information, or that it will perform the action called '0' (zero).

Similarly, on output PREFMAP-3 generates a special type of records, called *lines*, which have 132 positions if routed to a line printer, and sometimes 133 positions if displayed on a screen (the first extra character controls the line printer behavior). It is possible for PREFMAP-3 to write different pieces of output to different units, in the form of cards (again, not necessarily actual cards, depending on the type of output device). However, this is not assumed to be the case in the *standard output stream* as described in chapter 7. Cards and lines are discussed in groups called *blocks*, independently from the "blocks" that might be present in a file system; a number of cards or blocks together form a *deck*. The organization and interpretation of the symbols on a card is ruled by a *format*, positions are also called *columns*, and a group of columns a *field*. Constants given to the program are called *parameters*, whereas the statistical parameter estimates calculated by the program will always be called by their specific name, as defined in Part I of this User's Guide.

6.1 General set-up

The standard input stream consists of five blocks, which have a fixed, predetermined order (the order of cards within blocks is fixed as well). Schematically, we must have:

Title card	TITLE
Data specification card Analysis specification card Print/plot options card Unit number card	CONTROL BLOCK (6.3, 6.4, 6.5, 6.6)
Format card {Target configuration} cards	DATA BLOCK 1
Format card {External data matrix} cards	DATA BLOCK 2
{Option table} cards	MODEL OPTION BLOCK (6.2)

The *Title card* forms the first block. It simply identifies the job, and may contain any alphanumeric symbols (up to 80 characters). The PREFMAP-3 output will be labeled with this information.

Next, there must be a *Control block* formed by four cards. In the Control block all details of the job are specified, including information on the size and type of the three remaining blocks: Data block 1, Data block 2, and the Model option block. The *Model option block* contains specific information on the models to be fitted for each individual, or group of individuals. An understanding of its organization is needed for a proper specification of parameters in the Control block, and therefore the Model option block will be fully discussed first (section 6.2); after that, each card of the Control block is explained in a separate section (6.3–6.6).

Data block 1 must contain the *target configuration*. The first card is a format card, in particular a FORTRAN F-format card. Any of the standard FORTRAN specifications may be used freely, but in many cases the card will look like, e.g.,

(3F10.4)

indicating that each following target configuration card will contain three numbers of the *F*loating type, occupying 10 positions each, with a decimal point in the sixth position, so that there remain four positions for the fractional portion. After the format card there must follow m (number of objects) cards, each with p (number of dimensions) coordinate values. Any configuration can be given as input to the program. If the coordinates are not in column deviation form (column means equal to zero), PREFMAP-3 will perform a centering operation.

Data block 2 must contain the *external data matrix*. Like in Data block 1, the first card is a format card, and the earlier remarks about its construction apply here too. Next there must be n (number of individuals) cards, each with m (number of objects) dissimilarity values. Make sure that the scoring direction is: "small values ↔ high preferences" and "large value ↔ low preference", regardless of the model to be fitted. If the data happen to be coded in the other direction, a recoding facility *outside* PREFMAP-3 must be used. Most common is to subtract all values from the largest possible one, or to change all signs in the matrix.

Through specifications on the Unit number card of the Control block it is possible to change the standard input stream in the sense that PREFMAP-3 will read parts of the input from different sources (units). Either the target configuration, and/or the external data matrix, and/or the option table may be dropped from the standard input stream. Notice that the Title card, the Control block, and the two format cards *always* remain on the same, predetermined unit (with number 5, although it will mostly not be necessary for the user to specify this anywhere).

A concise description of the complete deck set-up is given in *Appendix B*, which provides sufficient guidance for the advanced user. The next sections implicitly refer to Appendix B; it is important to note that the order of description in consecutive sections *does not correspond* to the order of the blocks in the standard input stream (the Model option block is explained first; the Title and Data blocks have already been fully explained here).

6.2 The option table

To the large extent, the option table embodies the flexibility of PREFMAP-3; it monitors the variety of analyses to be done in a single run. The rows of the table are called *option sets*, its columns *analyses*, and each entry contains an *option*. An option, in this context, is a particular combination of a model from the PREFMAP hierarchy and a regression type. Since there are four possible models and three possible regression types, there are 12 options to choose from. They are characterized by two-letter acronyms, as shown in Table 6.1. Both an option set, and an analysis is a series of chosen options; the former refers to the things to be done for any individual, the latter refers to the presentation of results for a group of individuals. Let's first consider the situation in which there is only one individual.

In the case of a single individual the option table can have only one row (one option set), and at most four columns (four analyses). There is no restriction

Table 6.1: Acronyms used for model-regression combinations.

Model	Type of regression		
	*M*etric	Nonmetric *P*rimary approach	Nonmetric *S*econdary approach
*V*ector	VM	VP	VS
*U*nfolding	UM	UP	US
*W*eighted Unfolding	WM	WP	WS
*G*eneral Unfolding	GM	GP	GS

Table 6.2: Option table for four metric analyses.

option set	analysis			
	1	2	3	4
1	VM	UM	WM	GM

on the order of the options. Examples are given in Table 6.2, 6.3 and 6.4. Table 6.2 will inform PREFMAP-3 to successively apply the vector, the Unfolding, the weighted Unfolding, and the general Unfolding model all metrically.

Table 6.3 indicates that only one analysis has to be done, with the Unfolding model, nonmetrically with primary approach to ties (i.e., ties may become untied). Table 6.4 will cause PREFMAP-3 to perform three different unfolding analyses, first metrically, next nonmetrically with primary approach to ties, and finally nonmetrically with secondary approach to ties (i.e., ties must remain tied in the latter analysis). If more than four analyses on a single individual are desired, the data could be repeated and treated as a case of multiple individuals.

If there are n individuals in the external data matrix (as is most commonly the case), the following three possibilities can be distinguished:

(a) every individual gets a different option set;

(b) there are groups of individuals that share the same option set;

(c) every individual gets same option set.

The most general situation is (a); each individual becomes associated with one row of the option table. An example is given in Table 6.5, in which for every individual an entirely different series of options is specified. PREFMAP-3 executes the options in rowwise order: first all options of option set 1, next all options of option set 2, etc. After all rows have been processed (and interim results have been printed), the program considers all options in a column of the

Table 6.3: Option table for one single analysis.

option set	analysis			
	1	2	3	4
1	UP			

Table 6.4: Option table for three different Unfolding analyses.

option set	analysis 1	2	3	4
1	UM	UP	US	

Table 6.5: Option table, general type.

option set	analysis 1	2	3	4
1	VM	UM	GM	WM
2	VM	UM		
3	UM	UP		
4	UP			
5	UP		GP	
6	UP	US		WS
7	VP	US		

option table as a separate group, as one analysis. Clearly, the number of individuals in each analysis may be different. According to Table 6.5, all individuals are in analysis 1; PREFMAP-3 will give, upon request, a joint plot of the target configuration with vectors for individuals 1, 2, and 7, and ideal points for the others. Analysis 2 gives the unfolding results of individuals 1, 2, 3, 6, and 7; analysis 3 gives the general Unfolding results of 1 and 5; finally, analysis 4 gives the weighted unfolding results of 1 and 6. Notice that it is no problem to leave an intermediate entry in a row unspecified. For instance, when PREFMAP-3 reaches option set 5, it simply executes UP and GP consecutively; in analysis 2, it will give individual 5 coordinates zero (marked with "N.A.", not applied); the GP results of individual 5 are presented in analysis 3.

It is also not a problem to have *redundancies* in the option table. An example is Table 6.6: there is a horizontal redundancy in the last three option sets (VP under analysis 3 and 4), and a vertical one in rows (1, 2, 3), (4, 5, 6, 7) and (8, 9, 10). Apparently, there are three groups of data (for instance: preferences from male subjects, preferences from female subjects, and a number of properties characterizing the objects). Analysis 1, 2, and 3 each focus on a separate group while analysis 4 gives a joint representation of all groups under the same model. The horizontal redundancy will cause PREFMAP-3 to compute VP twice in the last group (somewhat superfluously, indeed, but no harm is done). The vertical redundancy is in fact possibility (b) mentioned above, and can be communicated to PREFMAP-3 in a more economical way: through a proper specification on the first card of the Control block (see section 6.3), Table 6.7 will be sufficient information for PREFMAP-3 to be able to execute each option set repeatedly for all individuals within a group. The reduced form of the table explains the introduction of the term option set: a row may correspond with one individual, or with a group. Obviously, possibility (c) — every individual gets the same option set — is a special case of (b), and single-row tables like Table 6.2, 6.3 and 6.4 suffice. A very small number of distinct option sets (compared to the number of individuals) is most common in applications of PREFMAP-3. The

Table 6.6: Option table showing redundancies.

option set	analysis 1	2	3	4
1	UP			VP
2	UP			VP
3	UP			VP
4		UP		VP
5		UP		VP
6		UP		VP
7		UP		VP
8			VP	VP
9			VP	VP
10			VP	VP

Table 6.7: Reduced version of Table 6.6.

option set	analysis 1	2	3	4
1	UP			VP
2		UP		VP
3			VP	VP

possibility to have a short cut specification has been limited to the range of one up to four groups. If more than four groups of individuals share the same options, the user will still have to specify a complete option table.

The option table must be coded in the Model option block with one card for each option set. Each two-letter acronym must occupy the first two positions of four-column fields. For example:

```
         1111111    column number
1234567890123456
UM  WM  GM  VM      option codes
```

Empty cells of the option table must be coded as blanks; there are no default options. The fact that the option acronyms have to link up to the left might be confusing for some users, as it is in contrast to the usual FORTRAN I-format convention. The options may be coded in either upper to lower case characters.

6.3 Data specifications

The data specification card is the first card of the Control block, and may contain from 3 up to 7 parameters, coded in five-column fields (linked up to the right). The parameters will each be described in turn.

The first parameter indicates the *number of rows* in the external data matrix, i.e. the number of individuals n. It has no default value.

The second parameter indicates the *number of columns* in the external data matrix, i.e. the number of objects m. It corresponds to the number of rows in the target configuration, and has no default value.

The third parameter indicates the *option set selection*: it describes the way in which option sets in the option table are to be linked to the individuals. If it is given value 0 (zero), PREFMAP-3 will apply the options in the first (and possibly only) row of the option table to each and every individual. The value 1 designates the situation that all individuals are to be analyzed with a different option set; thus the program will expect an option table with n rows. The value 2 tells PREFMAP-3 that some individuals should have the same option set applied to them, in a way to be specified through the next parameters.

The last four parameters are only needed when the third one (option set selection) has value 2 (subgroups with the same option set). Otherwise, they are ignored. Their values designate the *starting points* in the external data matrix for which a new option set applies. Thus each parameter must equal the row number of the first individual of each subgroup sharing the same option set. As an example, consider Table 6.6 as the complete specification for what has to be done with 10 individuals. Under option set selection 2 we can work with the reduced option Table 6.7, and have to give the 4th parameter value 1, the 5th parameter value 4, and the 6th parameter value 8. The maximum number of subgroups is four (hence at most four parameters). PREFMAP-3 expects an increasing sequence of values. Whenever the specified sequence is not increasing (e.g., the numbers 1, 5, 3, and 2 are given, in that order), the conflicting starting points (3 and 2) will be ignored, and the program acts as if a smaller number of subgroups has been specified (two subgroups, starting with 1 and 5). If the same rule by which PREFMAP-3 is able to determine the number of subgroups in general; e.g., " 1 4 8 0 " can only mean that there are three subgroups (therefore, no separate parameter for number of subgroups is needed).

The rows of the external data matrix must be in the right order according to the subgroups intended. If they are not, say we would like to apply the vector model and the Unfolding model alternatingly, we should specify that all rows have different option sets, and alternate a "vector card" with an "Unfolding card" in the Model option block.

6.4 Analysis specifications

The analysis specification card is the second card of the Control block, and contains 10 parameters. It controls the general characteristics of the analyses, and enables PREFMAP-3 to set up the right amount of working area. The first nine parameters must be coded as integer numbers in five-column fields, the last one is a floating point number occupying 10 positions.

The first parameter indicates the *number of dimensions* of the object space (p). It can be any number between 1 and $m-1$ (the number of objects minus one). Note that PREFMAP-3 will refuse to attempt specific analyses for which the number of free parameters (a function of p, see Table 3.1 in section 3.5) exceeds m. Also note that the program expects to be able to read p coordinate values from the target configuration cards in Data block 1 (which may contain more, but never less than that number of values).

The second parameter indicates the *maximum number of analyses in any option set*, or the number of columns in the option table. This need not be the same as the maximum number of options in any option set. For example, when using Tables 6.3, 6.4, 6.5 or 6.6, the value of this parameter must be 1, 3, 4, and 4, resp.

The third parameter controls the application of a *preliminary transformation of the configuration*. The default value (zero) will leave the target configuration unchanged. A value of 1 designates preliminary weighting of axes, a value of 2 designates preliminary rotation and rotation and weighting (the full transformation).

The fourth parameter controls the *standardization of the external data*. Since standardization does not affect the fit, and the individual data are more comparable when they have zero mean and unit variance, the data are standardized row-wise by default. In addition, it is possible to apply only centering (zero mean) by specifying 1, or only normalizing (sum of squares equal to m) by specifying 2. The value 3 designates no standardization at all.

The next four parameters indicate simply whether or not a model is going to be applied in any option set. This is necessary for efficient array allocation. When a model, say the Unfolding model, is not referred to on this card (by a '1'), but later on the program encounters an Unfolding model in some option set, the Unfolding model will not be applied, and a warning message will be printed. These parameters are ordered according to the complexity of the models:

- the fifth parameter: *application of the vector model*;
- the sixth parameter: *application of the Unfolding model*;
- the seventh parameter: *application of the weighted Unfolding model*;
- the eighth parameter: *application of the general Unfolding model*.

In all cases, a value of 1 designates the affirmative, and a 0 (zero) the negative specification.

The ninth parameter sets a *limit to the number of nonmetric iterations*. Obviously, it is needed only if there are nonmetric options specified. Its value is the maximum number of ALS cycles allowed in each nonmetric analysis (cf. chapter 5); the default value is 50. Usually PREFMAP-3 will be ready long before this number of iterations when the standard convergence criterion applies. If not, the maximum number of iterations generally has to be adjusted, because it is not a proper stopping rule but merely a safeguard.

The tenth parameter is the *convergence criterion*. for nonmetric iterations, needed to decide when to stop the ALS process. When the rate of change in the function (5.2) drops below the value of this parameter, the program concludes that the process has converged. The default value is .00001; with a more stringent criterion the number of iterations might increase beyond the limit of 50, the default value of the previous parameter. The user is advised against making the criterion more lenient, unless it is absolutely imperative to economize on computation time. The convergence criterion must be coded with format F10.8.

6.5 Print/plot options

The print/plot options card is the third card of the Control block, and contains 9 parameters of the five-column integer type. It controls the output of PREFMAP-3. A detailed description of the output will be given in chapter 7; described here is merely the way of getting various parts of it.

The first parameter controls how much of the *input data* is printed. By default the first 10 rows of the external data matrix (at most) are printed, to enable checking up on correct transfer. When a value of 2 is specified, the complete external data matrix is printed. The target configuration can be obtained by specifying 1 (to get it in addition to the first 10 rows of the external data), or 3 (to get it in addition to the complete external data).

The second parameter indicates what is to be printed *for each option*. Complete results (value 1) include the following:

- the metric fit and the variance accounted for;
- the nonmetric fit (when applicable);
- the coordinates of the vector or the ideal point;
- the normalized weights (when applicable);
- the orthogonal rotation matrix (when applicable);
- the criterion and predicted values;
- the slope and the intercept.

The terms criterion values and predicted values refer to the fact that PREFMAP-3 solves a regression problem. The criterion values either are the (possibly standardized) external data, or the external data transformed by a monotone function (for the nonmetric case). The predicted values are the best approximation of the criterion values under the chosen model (the right hand sides of the regression equations, including the slope and the intercept terms). The (nonmetric) fit is the correlation between the criterion values and the predicted values. The slope and intercept terms make it possible to construct the predicted values from the squared Euclidean distances between an ideal point and the object points. Part of the results can be suppressed by specifying a 2 (only the fit will be printed), or a 3 (fit as well as criterion and predicted values will be printed).

The third parameter controls the *selected results for each analysis*. An analysis is a series of models for different individuals brought together (as specified in the option table coded in the Model option block). A value of 1 will cause PREFMAP-3 to print a table of coordinate values, next a table of weights (if applicable), and finally a series of rotation matrices (if applicable). This is done for each analysis (column of the option table) in turn. Individuals are always identified with their original row number and the model that has been applied.

The fourth parameter indicates that a *scatter and transformation plot* must be made for each fitted model of the first N individuals; the parameter value N can be any integer in the range from 0 up to n, the number of individuals. In a single plot the criterion values are plotted against the data (the transformation), along with the predicted values (showing the vertical scatter around the transformation). Note that the individuals for whom the plots are to be obtained should come first in the data matrix. In metric analyses, the transformation will always be a straight line; in nonmetric analyses, it will be the optimal monotonically increasing step function. The scatter not only visualizes the fit, it provides in fact a pictorial breakdown of fit into its m components, each associated with one of the objects.

The fifth parameter calls for a *plot of ideal points* and/or *vectors* in the target configuration, again for each analysis in turn. The ideal points and vectors are labeled by integers $(1, 2, ..., 8, 9, 0, 1, ...)$, and the object points are labeled by characters (A,B,...,H,I,J,A,...). When the weighted Unfolding model or the general Unfolding model has been applied for more than one individual, the ideal point plot is followed by a plot of the weights. Further details are given in section 7.2. The value of this parameter, K, indicates that all pairs of the first K dimensions will be plotted. Thus if $K = 3$, then three plots are obtained: dimension 1 versus 2, 1 versus 3, and 2 versus 3. When $K = 2$, simply one plot is obtained; when $K = 1$, a two- dimensional plot of the first dimension is made (a straight line in a square box). Of course, K should not be larger than p, the dimensionality of the analysis.

The sixth parameter indicates printing of a *history of computation in the nonmetric regression* (1 = yes, 0 = no). When requested, a history is given for each nonmetric option. It will provide an impression of the course of the iterative process, which can be worthwhile looking at, especially when convergence is suspected to be slow or irregular.

The seventh parameter indicates computation and printing of the *F-statistic* (1 = yes, 0 = no). The F-ratios for significance testing will be printed for each model in an option set, while each model will also be compared to the nearest simpler model from the hierarchy available. This is done for each individual in turn. For example: when the weighted Unfolding model and the vector model have been applied, the F-statistics for testing the null hypotheses $\rho_W = 0, \rho_V = 0$, and $\rho_U = \rho_V$ are given (cf. section 3.5). In case the simple Unfolding model has been applied too, the program gives – in addition to the single model F-statistics – the F-ratios for the hypotheses $\rho_W = \rho_U$ and $\rho_U = \rho_V$. These F-ratios are always computed on the basis of the metric fit, and when nonmetric regressions were performed. In the latter case the F-statistics approach is no longer meaningful. When a model fits the data perfectly ($R_2 = 1$), the F-ratio is undefined and thus will not be given.

The last two parameters arrange *storing of the individual results* on an output device to be specified on the unit number card (section 6.6). The eighth parameter indicates storage of coordinates (code 1), of coordinates and weights (code 2), or of coordinates, weights and rotation matrices (code 3). When in operation, the results are always preceded by the target coordinates. The ninth parameter indicates storage of the external data together with the predicted values (code 1), or the external data together with the predicted values and the criterion values (code 2).

The prime usage of the storage facility is to be able to produce graphics on a plotting device outside PREFMAP-3. However, there can be another reason for using it. This is because the third and fifth parameter of the print/plot options card affect the efficiency of array allocation by the program itself. When a very large number of individuals must be dealt with, and the core memory area available to the program is limited, the storage facilities can be utilized as a substitute for printing. In general, then, it can be advised:

(a) When the number of individuals is small, to ask for

- complete results for each option (2nd parameter, especially if one is interested in the predicted values and the monotone transformation of the data);

- selected results for each analysis (3rd parameter);
- plotting of ideal points and/or vectors in the target configuration (5th parameter).

(b) When the number of individuals is large, to ask for

- the fit for each option only (2nd parameter);
- selected results for each analysis (3rd parameter);
- routing of individual results to other output units (8th and 9th parameter);
- plotting of ideal points and/or vectors in the target configuration (5th parameter).

(c) When a number of individuals is very large, to ask for

- routing of individual results to other output units (8th and 9th parameter).

If the default values for storage are used on the unit number card, PREFMAP-3 will actually print, rather than store, the individual results. Nevertheless, the program does not need additional array area for doing that, and the above-mentioned recommendations remain valid.

6.6 unit numbers for I/O blocks

The unit number card is the fourth card of the Control block, and contains 9 parameters of the five-column integer type. The first *three* parameters (with default value 5) all refer to *input units*, and can be used to alter the standard input stream (cf. section 6.1). The next *two* parameters (having no default value) refer to *scratch files*, which are needed during execution of the program. The last *four* parameters (with default value 6) refer to *output units* for receiving part of the results outside the standard output stream. See Appendix B for details. Whenever a unit number specification deviates from the default value, the user has to define a file in the operating system associated with the same number.

Note that the codes on the input files should satisfy the format specification given on the format cards, which always reside in the same unit as the parameter cards (section 6.1). If the target configuration, the external data and the option table share the same unit number, they should always be there in that order. It is also possible to have the target configuration and the option table on one unit, say 22, and the external data on another, say 21.

The required space for the two scratch files depends upon the size of the problem and the organizational features of the operating system.

Whether or not the output files are really used is controlled by the last two parameters of the print/plot options cards (section 6.5). Various partitions of the results are possible by specifying different output unit numbers. Since the results are written in the order of computation, we would otherwise obtain coordinates, weights, and rotation matrices alternatingly on the same output file, which would make them rather inaccessible for plotting afterwards.

6.7 Sample input stream

A sample input is given here to illustrate the complete input to PREFMAP-3 for a single run; the output of this small example will be discussed in section 7.3. The column numbers are given on top, for easy counting out.

```
    5    10   15   20   25   30   35   40   45   50   55
-------------------------------------------------------
*** TEST PREFMAP 3 ***
    5    5    2    1    3
    3    3   02    0    1    1    0    0   50    0
    3    1    1    2    2    1    1    0    0
    5    5    5    9    8    0    0    0    0
(8X,3F12.7)
           0.2863523   0.1391261  -0.4
           0.2459524  -0.0714838  -0.2
           0.0495586   0.1090570   0.0
          -0.1291228  -0.1841551   0.2
          -0.4527405   0.0074558   0.4
(5F7.3)
  1.500  3.500  1.500  1.500  3.000
  6.500  6.000  4.895  5.273  1.000
 -9.000 -7.677 -8.115 -7.625 -5.182
  3.000  3.000  3.000  3.000  3.000
  9.000  8.667  8.077  8.375  8.000
VM  VP
VS  UP  WM
-------------------------------------------------------
```

Chapter 7

Description of output

7.1 General output structure

The standard output stream consists of three major blocks, with contents highly dependent upon the specification of the print/plot options card. Schematically, we get:

Heading Overview of chosen parameters Report of data & options	JOB INFORMATION BLOCK
Results for individual 1 ⋮ Results for individual i ⋮ Results for individual n Summary of results	INDIVIDUAL MODELS BLOCK
Summary tables for consecutive analyses Joint plots for consecutive analyses	ANALYSIS BLOCK

The output routed to other units has a form comparable to the Individual models block, and will actually be inserted there if the default unit numbers are in operation.

7.2 Overview of the output blocks

The *Job information block* is the least elastic part of the output; it is only the report of the data that can be compressed (first parameter of the print/plot options card). The overview of chosen parameters is given in two forms; as an *echo* of the parameters cards, enabling the user to verify the literal input instructions, and as a *list of interpreted instructions* providing the user with feedback on the actions actually selected. The subheadings of the list are numbered with the corresponding card numbers printed in form of the echo. When the target configuration turns out not to be centered at the mean, this is remedied by the program. The mean for each original dimension will be printed. When a preliminary transformation has been requested (third parameter of the analysis specification card) there will always be a print of the transformation target configuration, as well as the orthogonal rotation matrix and/or the normalized weights.

After the job information block PREFMAP-3 starts giving results for each individual in turn, across options, except when *all* parameters on the print/plot options card have default values. In the latter case only the last part of the *Individual models block* is given: a summary of results consisting of the *average fit* across individuals sharing the same option (for each of the options present, not necessarily in the same column of the option table). For the metric options the total variance, the total variance accounted for, and the proportion of total variance accounted for are printed as well. The user is referred to section 6.5 for an explanation of the selections of individual results possible; here merely a number of additional details are mentioned.

The vector coordinates are normalized such that their length is proportional to the individual fit. Their overall size is determined by the size of the target configuration. For each Unfolding model it is indicated whether the fitted point is an ideal or an anti-ideal point. Saddle points are recognized by inspection of the sign pattern of the fitted weights. When a coordinate value "9999.0" is printed for an ideal point this indicates that the weight for the squared predictor term in the regression equation (cf. chapter 3) has become almost zero. In such a case the ideal point is located at infinity (or is behaving like a vector, one could say). Thus it might be wise to reanalyse the individual with the vector model. For the weighted models, the weights are normalized such that their sum squares equals the number of dimensions.

All parts of the *Analysis block* are optional. The summary tables for consecutive analyses are controlled by the third parameter of the print/plot options card, the joint plots by the fifth parameters on that card. Firstly all coordinates for the first analysis (e.g., the general Unfolding model for each individual) are given, followed by individual normalized weights and rotation matrices. These results are printed next to the row number and the model that has been applied (especially useful for mixed models within an analysis). If an individual did not participate in an analysis, the coordinates will be given as "0.0 0.0" with the label "N.A." (not applied) as the option description. When plotted, such an individual is to be found in the origin, the natural point of inconspicuousness. After the results for analysis 1 the program continues with the results for subsequent analysis, if any.

In the joint plots for consecutive analyses the ideal points and vectors are labeled by integers (1,2,...,8,9,0,1,...) and the object points are labeled by characters (A,B ...,H,I,J,A,...). The convention to plot non-participating individuals in the origin is also maintained for fitted anti-ideal points or saddle points. These will be displayed in a separate plot to prevent interpretational confusion. In all cases the original row number is retained to label the points. When two or more points coincide, the location is labeled with "M" for "more points". As was mentioned before, an ideal point can be far outside the configuration of objects points (maximally "9999.0"). Including such a point in the joint plot would make it impossible to adequately display the other points. To prevent such a "degenerate" plot, all points that are far outside the range of the object space are omitted from the plot, and a warning is issued.

For the weighted and the general unfolding model the individual points are plotted in the so-called *common space*, i.e. the original target configuration. Individually reshaped plots could be made outside PREFMAP-3 by applying the appropriate (rotation and) weights to both the target configuration and the coordinates of the individual point. Finally, the weights are displayed in a separate plot. Under the general model we have to bear in mind that the weights pertain to differently oriented reference axes. We can still compare, however, the relative importance of preferred directions across individuals (e.g., individual 1 has a ratio of 2:1, whereas individual 2 has a ratio of 3:1 for the first axis against the second in an idiosyncratic orientation).

7.3 Sample output stream

This section will show the output of the program in response to the sample input stream from section 6.7. The example has not been designed to demonstrate a serious analysis. On the contrary, the example was created to show a number of possible peculiarities of which the user should be aware when using the program. The complete printout is presented at the end of this section.

In the first place the full preliminary transformation of the target configuration is asked for. This transformation cannot be performed: the number of target points should at least be 7 to fit the $p(p+1)/2 = 6$ parameters of the preliminary transformation in $p = 3$ dimensions. Next the input data is read, and the program detects that one row of data matrix has zero variance; hence this row will be omitted from the analysis. The other rows will maintain their original row number for easy identification.

When the program reads the option sets, where the first two rows of the data matrix share the same set, it finds the option WM in the second option set. However, the weighted Unfolding model cannot be applied: in the first place, there are too many parameters to be estimated $(2p+1 = 7)$, and, moreover, the weighted model has not been referred to on the analysis specification card.

The results for individual 1 show the following peculiarities. The first option, the vector model applied metrically, gives a very poor fit (.306). This result is depicted in the accompanying scatter plot, where the points for the criterion values (labeled by a star) and the predicted values (labeled by a "D") are far apart. When the vector model is next applied nonmetrically, the fit proves considerably (.985); but the criterion values, which are the external data transformed by a monotone function, are hardly informative. There remain only two distinct values; -2.0 (the star in the lower left corner of the plot) and 0.5 (the "M" in the upper left corner, in this case indicating two stars on the right). The fact that the "D" points are relatively close to the stars is reflected in the relatively high correlation. One should always mistrust such a dramatic difference in fit between the metric and the nonmetric application of a model. The analysis for individual 2 shows a rather high fit for the metric vector model (.961). When next the nonmetric model is applied, the fit is perfect, at the cost of obtaining a three-step transformation shown in the corresponding scatter/transformation plot. The "M" labels indicate that the criterion values and the predicted values coincide completely.

For individual 3 the metric Unfolding model fits perfectly. Hence there is no improvement possible for the nonmetric model, and the F-statistics are not defined. The same applies to the analysis for individual 5, and these results could already have been anticipated since the number of target points equals exactly the number of parameters that has to be estimated ($p + 2 = 5$). When we inspect the selected results across analyses, we see that for individual 4 zeros have been assigned to the coordinates and weights, while for analysis 3 all individuals obtain zeros (since 1 and 2 were not included in this analysis, and the option "WM" for 3 and 5 could not be applied).

The first joint plot of the target points and the vectors from analysis 1 shows the coordinates for individual 4 plotted in the origin ("M"). In the joint plot for analysis 2, which shows vectors for individuals 1 and 2, and an ideal point for individual 3, we do not find the ideal point for individual 5. Because individual 5 obtained negative weights under the Unfolding model, the point should be depicted in a separate plot as an anti-ideal point. However, we do not actually obtain this plot, because the coordinates of the anti-ideal point are too much outside the range of the target coordinates.

```
P R E F M A P - 3
                                  *** EXTERNAL UNFOLDING ***               JACQUELINE MEULMAN
 VERSION - 1.0                    ***          &          ***               WILLEM HEISER
 FEBRUARY 1985                    *** PROPERTY FITTING   ***               J. DOUGLAS CARROLL
                                                                           BELL LABORATORIES
                                                                           MURRAY HILL, NJ


1 JOB TITLE: *** TEST PREFMAP 3 ***


ECHO OF PARAMETER CARDS:


2    5    5    2    1    3    0    0
3    3    3    2    0    1    1    0    0   500.00000000
4    3    1    1    2    2    1    1    0    0
5    5    5    5    9    8    0    0    0    0


2 DATA SPECIFICATIONS:

    NUMBER OF ROW POINTS (THESE ARE FITTED) IS      5
    NUMBER OF COLUMN POINTS (THESE ARE GIVEN) IS    5
    OPTION SET SELECTION:                           2
             0 = ALL ROWS SAME OPTION SET
             1 = ALL ROWS DIFFERENT OPTION SETS
             2 = SPECIFIED ROWS SAME OPTION SET
    OPTION SETS START WITH ROWS                     1    3


3 ANALYSIS SPECIFICATIONS:

    THE NUMBER OF DIMENSIONS IS                     3
    MAXIMUM NUMBER OF ANALYSES IN ANY OPTION SET    3
    PRELIMINARY TRANSFORMATION OF CONFIGURATION     2
             0 = REMAINS UNCHANGED
             1 = WEIGHTED
             2 = ROTATED AND WEIGHTED
    STANDARDIZE EXTERNAL DATA                       0
             0 = YES (=1+2)
             1 = CENTER ONLY
             2 = NORMALIZE ONLY
             3 = NONE OF THE ABOVE
    MODELS:  0 = NOT APPLIED, 1 = APPLIED
                 VECTOR MODEL                       1
                 UNFOLDING MODEL                    1
                 WEIGHTED UNFOLDING MODEL           0
                 GENERAL UNFOLDING MODEL            0
    THE NUMBER OF NON-METRIC ITERATIONS IS         50
    THE CONVERGENCE CRITERION IS             0.10E-04


4 PRINT/PLOT OPTIONS: 0 = NONE

    PRINT INPUT:                                    3
             1 = TARGET CONFIGURATION
             2 = EXTERNAL DATA
             3 = 1 & 2
    PRINT RESULTS ACROSS OPTION SETS                1
             1 = COMPLETE RESULTS
             2 = FIT
             3 = 2 & CRITERION-PREDICTED VALUES
    SELECTED RESULTS ACROSS ANALYSES                1
    SCATTER&TRANSFORMATION PLOT FOR N ROWS,  N =    2
    PAIRWISE PLOTS IDEAL POINTS/VECTORS,   DIM =    2
    HISTORY OF NON-METRIC REGRESSION                1
    STATISTICS FOR METRIC ANALYSES                  1
    STORE OUTPUT:                                   0
             1 = COORDINATES
             2 = LIKE 1 + WEIGHTS
             3 = LIKE 2 + ROTATION MATRICES
    STORE OUTPUT:                                   0
             1 = PREDICTED VALUES
             2 = LIKE 1 + CRITERION
```

```
5 UNIT NUMBERS FOR INPUT/OUTPUT

    THE CONFIGURATION WILL BE READ FROM UNIT        5
    THE EXTERNAL DATA WILL BE READ FROM UNIT        5
    THE OPTIONS WILL BE READ FROM UNIT              5
    UNIT NUMBER FOR SCRATCH FILE 1                  9
    UNIT NUMBER FOR SCRATCH FILE 2                  8
    OUTPUT UNIT FOR THE COORDINATES                 0
    OUTPUT UNIT FOR THE WEIGHTS                     0
    OUTPUT UNIT FOR THE ROTATION MATRICES           0
    OUTPUT UNIT FOR PREDICTED&CRITERION VALUES      0



*** WARNING *** THE PRELIMINARY TRANSFORMATION WILL NOT BE PERFORMED, BECAUSE THE
                NUMBER OF TARGET POINTS IS NOT SUFFICIENT.


THE CONFIGURATION WILL BE READ WITH FORMAT (8X,3F12.7)

THE TARGET CONFIGURATION WILL BE CENTERED: MEAN ORIGINAL DIMENSION    1      0.000
THE TARGET CONFIGURATION WILL BE CENTERED: MEAN ORIGINAL DIMENSION    2      0.000
THE TARGET CONFIGURATION WILL BE CENTERED: MEAN ORIGINAL DIMENSION    3      0.000


TARGET CONFIGURATION (CENTERED)
-------------------------------
             1        2        3

     1      0.286    0.139   -0.400
     2      0.246   -0.071   -0.200
     3      0.050    0.109    0.000
     4     -0.129   -0.184    0.200
     5     -0.453    0.007    0.400


THE EXTERNAL DATA WILL BE READ WITH FORMAT (5F7.3)



    5 ROWS OF THE EXTERNAL DATA
-------------------------------
             1        2        3        4        5
     1      1.500    3.500    1.500    1.500    3.000
     2      6.500    6.000    4.895    5.273    1.000
     3     -9.000   -7.677   -8.115   -7.625   -5.182
     4      3.000    3.000    3.000    3.000    3.000

ROW    4 WILL BE OMITTED FROM THE ANALYSIS SINCE IT HAS ZERO VARIANCE


     5      9.000    8.667    8.077    8.375    8.000

MODEL:       V  = VECTOR
             U  = UNFOLDING
             W  = WEIGHTED UNFOLDING
             G  = GENERAL UNFOLDING
REGRESSION:  M = METRIC
              P = NON-METRIC, PRIMARY APPROACH TO TIES
              S = NON-METRIC, SECONDARY APPROACH TO TIES


               ANALYSIS
OPTION SET:   1   2   3
         1  VM  VP
         2  VS  UP  WM

*** WARNING *** OPTION 3 WILL NOT BE APPLIED. EITHER THE MODEL HAS NOT BEEN REFERRED TO
                ON CARD 3 OR THE NUMBER OF TARGET POINTS IS NOT SUFFICIENT TO FIT THIS MODEL.
```

```
ROW=    1 ANALYSIS= 1 VECTOR MODEL              METRIC REGRESSION
=================================================================

           METRIC FIT          0.306  VARIANCE      1.000    V.A.F.      0.094

           COORDINATES
           -----------

             1        2        3
     1      0.108    0.112    0.100

           CRITERION VALUES (EXTERNAL DATA, POSSIBLY TRANSFORMED BY MONOTONE FUNCTION)
           ----------------

             1        2        3        4        5
     1     -0.803    1.491   -0.803   -0.803    0.918

           PREDICTED VALUES
           ----------------

             1        2        3        4        5
     1     -0.176    0.040   -0.480    0.397    0.219

           SLOPE   27.33366   INTERCEPT    0.00000
```

```
SCATTER PLOT. DATA (X-AXIS) VS CRITERION (*) AND PREDICTED VALUES (D) (M=*+D), ROW  1 VECTOR MODEL
            .--+------+------+------+------+------+------+------+------+------+------+---.
      1.491 I                                                                        *   I
      1.450 I                                                                            I
      1.408 I                                                                            I
      1.367 I                                                                            I
      1.325 I                                                                            I
      1.283 I                                                                            I
      1.242 I                                                                            I
      1.200 I                                                                            I
      1.159 I                                                                            I
      1.117 I                                                                            I
      1.076 I                                                                            I
      1.034 I                                                                            I
      0.993 I                                                                            I
      0.951 I                                                                            I
      0.909 I                                                      *                     I
      0.868 I                                                                            I
      0.826 I                                                                            I
      0.785 I                                                                            I
      0.743 I                                                                            I
      0.702 I                                                                            I
      0.660 I                                                                            I
      0.619 I                                                                            I
      0.577 I                                                                            I
      0.535 I                                                                            I
      0.494 I                                                                            I
      0.452 I                                                                            I
      0.411 I  D                                                                         I
      0.369 I                                                                            I
      0.328 I                                                                            I
      0.286 I                                                                            I
      0.245 I                                                                            I
      0.203 I                                                      D                     I
      0.162 I                                                                            I
      0.120 I                                                                            I
      0.078 I                                                                            I
      0.037 I                                                                        D   I
     -0.005 I                                                                            I
     -0.046 I                                                                            I
     -0.088 I                                                                            I
     -0.129 I                                                                            I
     -0.171 I  D                                                                         I
     -0.212 I                                                                            I
     -0.254 I                                                                            I
     -0.296 I                                                                            I
     -0.337 I                                                                            I
     -0.379 I                                                                            I
     -0.420 I                                                                            I
     -0.462 I  D                                                                         I
     -0.503 I                                                                            I
     -0.545 I                                                                            I
     -0.586 I                                                                            I
     -0.628 I                                                                            I
     -0.670 I                                                                            I
     -0.711 I                                                                            I
     -0.753 I                                                                            I
     -0.794 I  M                                                                         I
            .--+------+------+------+------+------+------+------+------+------+------+---.
            -0.803 -0.574 -0.344 -0.115  0.115  0.344  0.574  0.803  1.032  1.262  1.491
```

```
ROW=  1 ANALYSIS= 2 VECTOR MODEL        NON-METRIC REGRESSION    PRIMARY APPROACH TO TIES
=========================================================================================

          METRIC FIT          0.306  VARIANCE      1.000    V.A.F.      0.094

              HISTORY OF NON-METRIC REGRESSION
              --------------------------------
                                            DIFFERENCE WITH
              ITERATION       FIT           PRECEDING ITERATION
                       1      0.92946       0.62341
                       2      0.94096       0.01150
                       3      0.94956       0.00860
                       4      0.95847       0.00891
                       5      0.96705       0.00858
                       6      0.97473       0.00769
                       7      0.98120       0.00646
                       8      0.98526       0.00406
                       9      0.98549       0.00023
                      10      0.98549       0.00000

          NON-METRIC FIT      0.985

          COORDINATES
          -----------

            1        2        3
     1    0.369    0.258    0.390

          CRITERION VALUES (EXTERNAL DATA, POSSIBLY TRANSFORMED BY MONOTONE FUNCTION)
          ----------------

            1        2        3        4        5
     1    0.500    0.500   -2.000    0.500    0.500

          PREDICTED VALUES
          ----------------

            1        2        3        4        5
     1    0.612    0.241   -1.942    0.714    0.375

          SLOPE   41.88269   INTERCEPT    0.00000
```

```
SCATTER PLOT. DATA (X-AXIS) VS CRITERION (*) AND PREDICTED VALUES (D) (M=*+D), ROW  1 VECTOR MODEL
            .--+------+------+------+------+------+------+------+------+------+------+---.
      0.714 I       D                                                                    I
      0.665 I                                                                            I
      0.616 I       D                                                                    I
      0.566 I                                                                            I
      0.517 I       M                                           *              *         I
      0.468 I                                                                            I
      0.419 I                                                                            I
      0.370 I                                                   D                        I
      0.321 I                                                                            I
      0.272 I                                                                            I
      0.222 I                                                                  D         I
      0.173 I                                                                            I
      0.124 I                                                                            I
      0.075 I                                                                            I
      0.026 I                                                                            I
     -0.023 I                                                                            I
     -0.073 I                                                                            I
     -0.122 I                                                                            I
     -0.171 I                                                                            I
     -0.220 I                                                                            I
     -0.269 I                                                                            I
     -0.318 I                                                                            I
     -0.367 I                                                                            I
     -0.417 I                                                                            I
     -0.466 I                                                                            I
     -0.515 I                                                                            I
     -0.564 I                                                                            I
     -0.613 I                                                                            I
     -0.662 I                                                                            I
     -0.712 I                                                                            I
     -0.761 I                                                                            I
     -0.810 I                                                                            I
     -0.859 I                                                                            I
     -0.908 I                                                                            I
     -0.957 I                                                                            I
     -1.006 I                                                                            I
     -1.056 I                                                                            I
     -1.105 I                                                                            I
     -1.154 I                                                                            I
     -1.203 I                                                                            I
     -1.252 I                                                                            I
     -1.301 I                                                                            I
     -1.351 I                                                                            I
     -1.400 I                                                                            I
     -1.449 I                                                                            I
     -1.498 I                                                                            I
     -1.547 I                                                                            I
     -1.596 I                                                                            I
     -1.646 I                                                                            I
     -1.695 I                                                                            I
     -1.744 I                                                                            I
     -1.793 I                                                                            I
     -1.842 I                                                                            I
     -1.891 I                                                                            I
     -1.940 I       D                                                                    I
     -1.990 I       *                                                                    I
            .--+------+------+------+------+------+------+------+------+------+------+---.
            -1.013 -0.741 -0.470 -0.199  0.073  0.344  0.616  0.887  1.158  1.430  1.701
```

```
STATISTICS ACROSS OPTIONS FOR ROW    1
--------------------------------------
THE STATISTICS GIVEN BELOW DEPEND ONLY ON THE METRIC FIT, NOT ON THE NON-METRIC FIT.

VECTOR MODEL,       P.VAF. = 0.094, F =   0.034 WITH   3 AND   1 DEGREES OF FREEDOM
```

```
ROW=    2 ANALYSIS= 1 VECTOR MODEL              METRIC REGRESSION
=================================================================

          METRIC FIT          0.961  VARIANCE      1.000    V.A.F.      0.923

          COORDINATES
          -----------

            1        2        3
     2    -0.520    0.250   -0.060

          CRITERION VALUES (EXTERNAL DATA, POSSIBLY TRANSFORMED BY MONOTONE FUNCTION)
          ----------------

            1        2        3        4        5
     2     0.907    0.650    0.083    0.277   -1.916

          PREDICTED VALUES
          ----------------

            1        2        3        4        5
     2     0.724    1.074   -0.012   -0.073   -1.712

          SLOPE    8.02532   INTERCEPT    0.00000
```

```
SCATTER PLOT. DATA (X-AXIS) VS CRITERION (*) AND PREDICTED VALUES (D) (M=*+D), ROW  2 VECTOR MODEL
            .--+------+------+------+------+------+------+------+------+------+------+---.
      1.074 I                                                                D           I
      1.020 I                                                                            I
      0.965 I                                                                            I
      0.911 I                                                                      *     I
      0.857 I                                                                            I
      0.803 I                                                                            I
      0.749 I                                                                      D     I
      0.695 I                                                                            I
      0.640 I                                                                *           I
      0.586 I                                                                            I
      0.532 I                                                                            I
      0.478 I                                                                            I
      0.424 I                                                                            I
      0.370 I                                                                            I
      0.316 I                                                                            I
      0.261 I                                                       *                    I
      0.207 I                                                                            I
      0.153 I                                                                            I
      0.099 I                                                  *                         I
      0.045 I                                                                            I
     -0.009 I                                                  D                         I
     -0.064 I                                                       D                    I
     -0.118 I                                                                            I
     -0.172 I                                                                            I
     -0.226 I                                                                            I
     -0.280 I                                                                            I
     -0.334 I                                                                            I
     -0.388 I                                                                            I
     -0.443 I                                                                            I
     -0.497 I                                                                            I
     -0.551 I                                                                            I
     -0.605 I                                                                            I
     -0.659 I                                                                            I
     -0.713 I                                                                            I
     -0.768 I                                                                            I
     -0.822 I                                                                            I
     -0.876 I                                                                            I
     -0.930 I                                                                            I
     -0.984 I                                                                            I
     -1.038 I                                                                            I
     -1.093 I                                                                            I
     -1.147 I                                                                            I
     -1.201 I                                                                            I
     -1.255 I                                                                            I
     -1.309 I                                                                            I
     -1.363 I                                                                            I
     -1.417 I                                                                            I
     -1.472 I                                                                            I
     -1.526 I                                                                            I
     -1.580 I                                                                            I
     -1.634 I                                                                            I
     -1.688 I   D                                                                        I
     -1.742 I                                                                            I
     -1.797 I                                                                            I
     -1.851 I                                                                            I
     -1.905 I   *                                                                        I
            .--+------+------+------+------+------+------+------+------+------+------+---.
            -2.000 -1.701 -1.402 -1.103 -0.804 -0.505 -0.206  0.093  0.392  0.691  0.990
```

```
ROW=  2 ANALYSIS= 2 VECTOR MODEL        NON-METRIC REGRESSION    PRIMARY APPROACH TO TIES
=========================================================================================

           METRIC FIT          0.961  VARIANCE      1.000   V.A.F.      0.923

               HISTORY OF NON-METRIC REGRESSION
               --------------------------------
                                              DIFFERENCE WITH
               ITERATION      FIT             PRECEDING ITERATION
                        1      0.99313        0.03253
                        2      0.99790        0.00478
                        3      0.99934        0.00143
                        4      0.99979        0.00045
                        5      0.99993        0.00014
                        6      0.99998        0.00005
                        7      0.99999        0.00001
                        8      1.00000        0.00000

           NON-METRIC FIT      1.000

           COORDINATES
           -----------
             1        2        3
    2      -0.247    0.405    0.374

           CRITERION VALUES (EXTERNAL DATA, POSSIBLY TRANSFORMED BY MONOTONE FUNCTION)
           ----------------
             1        2        3        4        5
    2       1.034    1.034   -0.200   -0.200   -1.667

           PREDICTED VALUES
           ----------------
             1        2        3        4        5
    2       1.033    1.037   -0.201   -0.202   -1.666

           SLOPE    6.29833   INTERCEPT    0.00000
```

```
SCATTER PLOT. DATA (X-AXIS) VS CRITERION (*) AND PREDICTED VALUES (D) (M=*+D), ROW  2 VECTOR MODEL
       .--+------+------+------+------+------+------+------+------+------+------+---.
 1.096 I                                                                            I
 1.045 I                                                                 M      M   I
 0.994 I                                                                            I
 0.943 I                                                                            I
 0.891 I                                                                            I
 0.840 I                                                                            I
 0.789 I                                                                            I
 0.738 I                                                                            I
 0.687 I                                                                            I
 0.636 I                                                                            I
 0.585 I                                                                            I
 0.534 I                                                                            I
 0.482 I                                                                            I
 0.431 I                                                                            I
 0.380 I                                                                            I
 0.329 I                                                                            I
 0.278 I                                                                            I
 0.227 I                                                                            I
 0.176 I                                                                            I
 0.125 I                                                                            I
 0.073 I                                                                            I
 0.022 I                                                                            I
-0.029 I                                                                            I
-0.080 I                                                                            I
-0.131 I                                                                            I
-0.182 I                                                   M    M                   I
-0.233 I                                                                            I
-0.285 I                                                                            I
-0.336 I                                                                            I
-0.387 I                                                                            I
-0.438 I                                                                            I
-0.489 I                                                                            I
-0.540 I                                                                            I
-0.591 I                                                                            I
-0.642 I                                                                            I
-0.694 I                                                                            I
-0.745 I                                                                            I
-0.796 I                                                                            I
-0.847 I                                                                            I
-0.898 I                                                                            I
-0.949 I                                                                            I
-1.000 I                                                                            I
-1.051 I                                                                            I
-1.103 I                                                                            I
-1.154 I                                                                            I
-1.205 I                                                                            I
-1.256 I                                                                            I
-1.307 I                                                                            I
-1.358 I                                                                            I
-1.409 I                                                                            I
-1.460 I                                                                            I
-1.512 I                                                                            I
-1.563 I                                                                            I
-1.614 I                                                                            I
-1.665 I  M                                                                         I
-1.716 I                                                                            I
       .--+------+------+------+------+------+------+------+------+------+------+---.
       -1.916 -1.634 -1.352 -1.069 -0.787 -0.505 -0.223  0.060  0.342  0.624  0.907
```

```
STATISTICS ACROSS OPTIONS FOR ROW    2
--------------------------------------
THE STATISTICS GIVEN BELOW DEPEND ONLY ON THE METRIC FIT, NOT ON THE NON-METRIC FIT.

VECTOR MODEL,       P.VAF. = 0.923, F =   3.981 WITH   3 AND   1 DEGREES OF FREEDOM
```

```
ROW=  3 ANALYSIS= 1 VECTOR MODEL        NON-METRIC REGRESSION  SECONDARY APPROACH TO TIES
=========================================================================================

          METRIC FIT          0.898  VARIANCE      1.000    V.A.F.      0.807

              HISTORY OF NON-METRIC REGRESSION
              --------------------------------
                                         DIFFERENCE WITH
              ITERATION       FIT        PRECEDING ITERATION
                       1      0.98603    0.08762
                       2      0.99826    0.01223
                       3      0.99979    0.00153
                       4      0.99997    0.00019
                       5      1.00000    0.00002
                       6      1.00000    0.00000

          NON-METRIC FIT      1.000

          COORDINATES
          -----------

            1        2        3
     3     0.488    0.224    0.277

          CRITERION VALUES (EXTERNAL DATA, POSSIBLY TRANSFORMED BY MONOTONE FUNCTION)
          ----------------

            1        2        3        4        5
     3    -0.898   -0.724   -0.724    0.728    1.617

          PREDICTED VALUES
          ----------------

            1        2        3        4        5
     3    -0.897   -0.725   -0.724    0.729    1.617

          SLOPE   14.88134   INTERCEPT    0.00000
```

```
ROW=  3 ANALYSIS= 2 UNFOLDING MODEL     NON-METRIC REGRESSION    PRIMARY APPROACH TO TIES
=========================================================================================

          METRIC FIT          1.000  VARIANCE      1.000    V.A.F.      1.000

              HISTORY OF NON-METRIC REGRESSION
              --------------------------------
                                              DIFFERENCE WITH
              ITERATION      FIT              PRECEDING ITERATION

                      1      1.00000          0.00000

          NON-METRIC FIT      1.000

          COORDINATES IDEAL POINT
          -----------------------

            1        2        3
     3    -0.891   -0.162   -0.903

          CRITERION VALUES (EXTERNAL DATA, POSSIBLY TRANSFORMED BY MONOTONE FUNCTION)
          ----------------

            1        2        3        4        5
     3    -1.167   -0.124   -0.469   -0.083    1.843

          PREDICTED VALUES
          ----------------

            1        2        3        4        5
     3    -1.167   -0.124   -0.469   -0.083    1.843

          SLOPE   15.93727   INTERCEPT  -28.74530

       STATISTICS ACROSS OPTIONS FOR ROW    3
       --------------------------------------
       THE STATISTICS GIVEN BELOW DEPEND ONLY ON THE METRIC FIT, NOT ON THE NON-METRIC FIT.

       PERFECT FIT FOR OPTION 2. NO STATISTICS.

       VECTOR MODEL,       P.VAF. = 0.807, F =   1.395 WITH   3 AND   1 DEGREES OF FREEDOM
```

```
ROW=  5 ANALYSIS= 1 VECTOR MODEL        NON-METRIC REGRESSION  SECONDARY APPROACH TO TIES
=========================================================================================

           METRIC FIT          0.997  VARIANCE      1.000    V.A.F.      0.994

               HISTORY OF NON-METRIC REGRESSION
               --------------------------------
                                            DIFFERENCE WITH
               ITERATION      FIT           PRECEDING ITERATION
                       1      1.00000       0.00285

                       2      1.00000       0.00000

           NON-METRIC FIT      1.000

           COORDINATES
           -----------

             1        2        3
     5      0.311    0.232    0.463

           CRITERION VALUES (EXTERNAL DATA, POSSIBLY TRANSFORMED BY MONOTONE FUNCTION)
           ----------------

             1        2        3        4        5
     5      1.502    0.770   -0.960   -0.227   -1.086

           PREDICTED VALUES
           ----------------

             1        2        3        4        5
     5      1.502    0.770   -0.960   -0.227   -1.086

           SLOPE   23.54395   INTERCEPT    0.00000

ROW=  5 ANALYSIS= 2 UNFOLDING MODEL     NON-METRIC REGRESSION    PRIMARY APPROACH TO TIES
=========================================================================================

           METRIC FIT          1.000  VARIANCE      1.000    V.A.F.      1.000

               HISTORY OF NON-METRIC REGRESSION
               --------------------------------
                                            DIFFERENCE WITH
               ITERATION      FIT           PRECEDING ITERATION
                       1      1.00000       0.00000

           NON-METRIC FIT      1.000

           COORDINATES ANTI-IDEAL POINT
           ----------------------------

             1        2        3
     5     -2.369   -1.185   -2.932

           CRITERION VALUES (EXTERNAL DATA, POSSIBLY TRANSFORMED BY MONOTONE FUNCTION)
           ----------------

             1        2        3        4        5
     5      1.547    0.653   -0.931   -0.131   -1.138
```

```
    PREDICTED VALUES
    ----------------

      1        2        3        4        5

 5    1.547    0.653   -0.931   -0.131   -1.138

    SLOPE    2.73786   INTERCEPT   43.19877


   STATISTICS ACROSS OPTIONS FOR ROW    5
   --------------------------------------
   THE STATISTICS GIVEN BELOW DEPEND ONLY ON THE METRIC FIT, NOT ON THE NON-METRIC FIT.

   PERFECT FIT FOR OPTION 2. NO STATISTICS.

   VECTOR MODEL,       P.VAF. = 0.994, F =  58.233 WITH   3 AND   1 DEGREES OF FREEDOM



SUMMARY OF RESULTS: (PROPORTION OF TOTAL VARIANCE ACCOUNTED FOR ONLY GIVEN FOR METRIC OPTIONS)

MODEL '  VM  ' N =    2  AVERAGE FIT = 0.633
               ROOT MEAN SQUARED FIT = 0.713
TOTAL VARIANCE =    2.000   TOTAL VARIANCE ACCOUNTED FOR =     1.016
PROPORTION OF TOTAL VARIANCE ACCOUNTED FOR = 0.508

MODEL '  UP  ' N =    2  AVERAGE FIT = 1.000
               ROOT MEAN SQUARED FIT = 1.000

MODEL '  VP  ' N =    2  AVERAGE FIT = 0.993
               ROOT MEAN SQUARED FIT = 0.993

MODEL '  VS  ' N =    2  AVERAGE FIT = 1.000
               ROOT MEAN SQUARED FIT = 1.000


  ANALYSIS 1 COORDINATES
  ----------------------
                   1     VM      0.108    0.112    0.100
                   2     VM     -0.520    0.250   -0.060
                   3     VS      0.488    0.224    0.277
                   4     VS      0.000    0.000    0.000
                   5     VS      0.311    0.232    0.463


  ANALYSIS 2 COORDINATES
  ----------------------
                   1     VP      0.369    0.258    0.390
                   2     VP     -0.247    0.405    0.374
                   3     UP     -0.891   -0.162   -0.903
                   4     UP      0.000    0.000    0.000
                   5     UP     -2.369   -1.185   -2.932


  ANALYSIS 2 WEIGHTS
  ------------------
                   3     UP      1.000    1.000    1.000
                   4     UP      0.000    0.000    0.000
                   5     UP     -1.000   -1.000   -1.000


  ANALYSIS 3 COORDINATES
  ----------------------
                   1     N.A.    0.000    0.000    0.000
                   2     N.A.    0.000    0.000    0.000
                   3     N.A.    0.000    0.000    0.000
                   4     N.A.    0.000    0.000    0.000
                   5     N.A.    0.000    0.000    0.000
```

```
IDEAL POINTS AND/OR VECTORS (INTEGERS) IN TARGET CONFIGURATION. ANALYSIS 1 DIM 1 (X-AXIS) VS DIM 2 (Y-AXIS)
            .--+------+------+------+------+------+------+------+------+------+------+---.
      0.537 I                                                                            I
      0.519 I                                                                            I
      0.500 I                                                                            I
      0.482 I                                                                            I
      0.464 I                                                                            I
      0.446 I                                                                            I
      0.427 I                                                                            I
      0.409 I                                                                            I
      0.391 I                                                                            I
      0.373 I                                                                            I
      0.354 I                                                                            I
      0.336 I                                                                            I
      0.318 I                                                                            I
      0.300 I                                                                            I
      0.281 I                                                                            I
      0.263 I                                                                            I
      0.245 I  2                                                                         I
      0.226 I                                                           5            3   I
      0.208 I                                                                            I
      0.190 I                                                                            I
      0.172 I                                                                            I
      0.153 I                                                                            I
      0.135 I                                                         A                  I
      0.117 I                                         C   1                              I
      0.099 I                                                                            I
      0.080 I                                                                            I
      0.062 I                                                                            I
      0.044 I                                                                            I
      0.026 I                                                                            I
      0.007 I      E                               M                                     I
     -0.011 I                                                                            I
     -0.029 I                                                                            I
     -0.048 I                                                                            I
     -0.066 I                                                       B                    I
     -0.084 I                                                                            I
     -0.102 I                                                                            I
     -0.121 I                                                                            I
     -0.139 I                                                                            I
     -0.157 I                                                                            I
     -0.175 I                             D                                              I
     -0.194 I                                                                            I
     -0.212 I                                                                            I
     -0.230 I                                                                            I
     -0.248 I                                                                            I
     -0.267 I                                                                            I
     -0.285 I                                                                            I
     -0.303 I                                                                            I
     -0.322 I                                                                            I
     -0.340 I                                                                            I
     -0.358 I                                                                            I
     -0.376 I                                                                            I
     -0.395 I                                                                            I
     -0.413 I                                                                            I
     -0.431 I                                                                            I
     -0.449 I                                                                            I
     -0.468 I                                                                            I
            .--+------+------+------+------+------+------+------+------+------+------+---.
            -0.520 -0.420 -0.319 -0.218 -0.117 -0.016 0.085  0.186  0.286  0.387  0.488  .
```

```
IDEAL POINTS AND/OR VECTORS (INTEGERS) IN TARGET CONFIGURATION. ANALYSIS 2 DIM 1 (X-AXIS) VS DIM 2 (Y-AXIS)
            .--+------+------+------+------+------+------+------+------+------+------+---.
      0.740 I                                                                            I
      0.718 I                                                                            I
      0.695 I                                                                            I
      0.672 I                                                                            I
      0.649 I                                                                            I
      0.626 I                                                                            I
      0.603 I                                                                            I
      0.581 I                                                                            I
      0.558 I                                                                            I
      0.535 I                                                                            I
      0.512 I                                                                            I
      0.489 I                                                                            I
      0.467 I                                                                            I
      0.444 I                                                                            I
      0.421 I                                                                            I
      0.398 I                                     2                                      I
      0.375 I                                                                            I
      0.352 I                                                                            I
      0.330 I                                                                            I
      0.307 I                                                                            I
      0.284 I                                                                            I
      0.261 I                                                                        1   I
      0.238 I                                                                            I
      0.216 I                                                                            I
      0.193 I                                                                            I
      0.170 I                                                                            I
      0.147 I                                                                   A        I
      0.124 I                                                                            I
      0.101 I                                                      C                     I
      0.079 I                                                                            I
      0.056 I                                                                            I
      0.033 I                                                                            I
      0.010 I                          E                        M                        I
     -0.013 I                                                                            I
     -0.035 I                                                                            I
     -0.058 I                                                                            I
     -0.081 I                                                                 B          I
     -0.104 I                                                                            I
     -0.127 I                                                                            I
     -0.150 I                                                                            I
     -0.172 I  3                                                                         I
     -0.195 I                                            D                               I
     -0.218 I                                                                            I
     -0.241 I                                                                            I
     -0.264 I                                                                            I
     -0.287 I                                                                            I
     -0.309 I                                                                            I
     -0.332 I                                                                            I
     -0.355 I                                                                            I
     -0.378 I                                                                            I
     -0.401 I                                                                            I
     -0.423 I                                                                            I
     -0.446 I                                                                            I
     -0.469 I                                                                            I
     -0.492 I                                                                            I
     -0.515 I                                                                            I
            .--+------+------+------+------+------+------+------+------+------+------+---.
            -0.891 -0.765 -0.639 -0.513 -0.387 -0.261 -0.135 -0.009  0.117  0.243  0.369


  ANALYSIS 2 *** NOT PLOTTED ***   5   UP   -2.369   -1.185 *** SEVERELY OUT OF RANGE ***
```

Chapter 8

Some applications of PREFMAP-3

In this chapter three applications of PREFMAP-3 will be presented. because it is not possible to cover the whole range of applications here, the user might find it useful to consult one of the following applications: Coxon (1974), Cermak and Cornillon (1976), Davison and Jones (1976), Falbo (1977), Nygren and Jones (1977), Seligson (1977), Coxon and Jones (1978), Davison *et al.* (1980), and Kuyper (1980).

Heiser and De Leeuw (1981) have made an attempt to classify social science applications of preference mapping into prototypical groups. They distinguish:

- *general Thurstonian attitude scaling*: scale values for the attitude items are determined by a separate experimental procedure (cf. Torgerson (1958), chapter 4), and the problem of finding attitude scores for the individuals, given their list of "endorsements" of preference ratings, can be solved by PREFMAP (also if we switch from one-dimensional to multidimensional representations of the attitude items);

- *trade-off studies*: suppose we have a collection of objects known to differ on two negatively correlated desirable traits, e.g. a set of insurance policies varying in prize and in cover. We now may want to characterize customers in terms of their *safety bias* on the basis of reported preferences;

- *multidimensional psychophysics*: suppose we have a collection of objects selected as to differ on two physical attributes, e.g. a set of taste mixtures (say, alanine and glutamic acid combined in various concentrations), which are to be judges as to their *sweet-sourness*; or a set of odour mixtures (say, jasmine bergamot in various concentrations), to be judges on their *hedonic tone*;

- *multidimensional psychophysics*: here the objects are varied systematically on psychological attributes. Typically, one confronts subjects with hypothetical "stimulus persons", differing on, e.g., intelligence and dominance, and asks for a judgment of overall *likeableness*. A large amount of research has been dedicated to the discovery of the rule by which a subject combines different pieces of information into one final impression.

Table 8.1: Median rating scale values of 13 facial expressions.

	Expression	P-U	A-R	T-S
1.	Grief at death of mother	3.8	4.2	4.1
2.	Savoring a Coke	5.9	5.4	4.8
3.	Very pleasant surprise	8.8	7.8	7.1
4.	Maternal love – baby in arms	7.0	5.9	4.0
5.	Physical exhaustion	3.3	2.5	3.1
6.	Anxiety – something is wrong with her plane	3.5	6.1	6.8
7.	Anger at seeing dog beaten	2.1	8.0	8.2
8.	Physical strain – pulling hard on seat of chair	6.7	4.2	6.6
9.	Unexpectedly meets old boy friend	7.4	6.8	5.9
10.	Revulsion	2.9	3.0	5.1
11.	Extreme pain	2.2	2.2	6.4
12.	Knows her plane will crash (disaster)	1.1	8.6	8.9
13.	Light sleep	4.1	1.3	1.0

It will be clear that in all cases the prime advantage of the PREFMAP methodology is to be able to go beyond the assumption of monotonicity of the dependent variable with respect to the independent (i.e., varied or selected by the experimenter) variables. For a discussion of applying PREFMAP as a kind of quality control for Multidimensional Scaling and Internal Unfolding (study of *discriminant-, convergent-, or cross-validity*) the user is referred to Heiser and de Leeuw's original (1981) paper, and to Bechtel (1976, 1981).

8.1 Facial expressions

Our first application is a reanalysis of a classical example concerning facial expressions. The target configuration has been obtained by performing a nonmetric Multidimensional Scaling analysis of the dissimilarities collected by Abelson and Sermat (1962). The objects are selected photographs from the Lightfoot-series, in which an actress expresses 13 different emotions. They are listed in the stub of Table 8.1. The external data are the median ratings by 96 male undergraduates on three nine-point scales, obtained for the same 13 emotions by Engen *et al.* (1958), and reflecting judgments on the attributes "pleasant-unpleasant" (P-U), "attention-rejection" (A-R), and "tension-sleep" (T-S).

The data in Table 8.1 are to be interpreted dissimilarities with respect to the right pole of the bipolar attribute scales; e.g., emotion 12, disaster, is judged to be very unpleasant, and emotion 3, very pleasant surprise, is indeed judged to be very pleasant, i.e. very dissimilar to unpleasant. On the other hand, remember the convention in PREFMAP-3 to let a small dissimilarity correspond to a *large* projection score under the vector model (and to a *small* distance under the Unfolding model). For these data we consider the vector model as the most appropriate, because it most readily incorporates the bipolarity of the scales (viz., as two opposing directions along the vector). Only when the fit would be very poor we might consider to apply a more complex model, in which one of the poles would have to be favoured as the central one (to be mapped as the peak/dip of the preference function).

The data were analyzed both metrically and nonmetrically, and the resulting fit measures are collected in Table 8.2. Notice that the metric fit is already quite

Table 8.2: Fit measures for facial expressions, vector model.

Regression type	P-U	A-R	T-S
Metric	.961	.861	.947
Nonmetric	.988	.922	.991

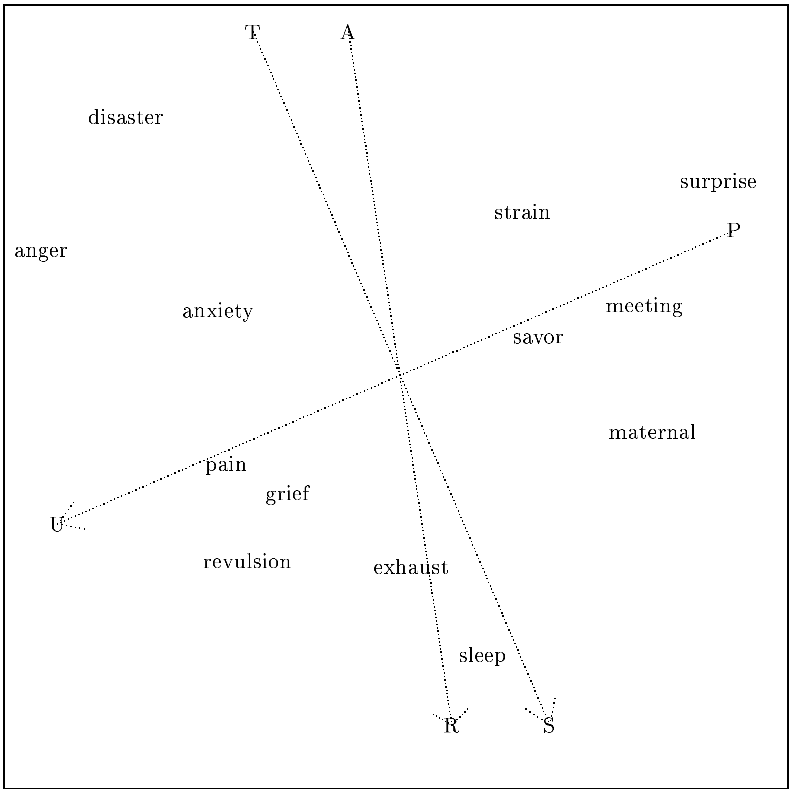

Figure 8.1: PREFMAP-3 solution for three attributes of facial expression

good for all attributes, and that it is only slightly improved by allowing an optimal monotone transformation. We may conclude that the attribute judgments are fairly coherent with the dissimilarity judgments, a conclusion also reached by Heiser and Meulman (1983), who analyzed the two sets of data simultaneously. The results for the present analysis are presented in Figure 8.1, which displays the points for the facial expressions and the vectors for the scales in a joint plot.

It is evident from Figure 8.1 that "attention-rejection" and "tension-sleep" partition the facial expressions in much the same way, and that both are psychologically independent from the "pleasant-unpleasant" contrast.

Table 8.3: Party allegiance of MPs in the Parliament 1972 study.

Party	Description	No. of MPs	MP-label
PSP	pacifist-socialist	2	P
PPR	radical	1	R
PvdA	labor	36	L
D'66	pragmatic liberal	8	6
ARP	protestant	11	A
KVP	catholic	32	K
CHU	protestant	10	U
VVD	conservative-liberal	15	V
GPV	calvinist	2	G
SGP	calvinist	1	S
DS'70	conservative-socialist	6	7

8.2 Parliament 1972

Parliament 1972[1] is a questionnaire study among the members of the Second Chamber of the Dutch Parliament (comparative with the House of Representatives in the U.S.). The members (MPs) expressed their preferences for the political parties residing in Parliament, and also their position with respect to 7 political issues on a nine-point scale. The political parties are given in Table 8.3, and the issues in Table 8.4. The objects in the PREFMAP-3 analysis are the Members, identified only by their party allegiance; the issues serve as the PREFMAP-3 individuals, thus the issue self-ratings are the external data of this application. To obtain a target the preference data have been analyzed with a metric *internal* Unfolding program (SMACOF3, Heiser and de Leeuw, 1979). From the SMACOF3 results the *coordinates for the MPs* served as the target configuration for the PREFMAP-3 analysis.

Since it can be argued that at least for some issues the vector model adage "the more the better" is not too suitable (more money to developing countries, but not the whole budget; taxes could be increased, but not to 100%) both the vector model and the Unfolding model have been tried. The fit measures are collected in Table 8.5.

On the basis of the fit measures we would decide that the vector vector is yet the more appropriate model; the Unfolding model, either metrically or nonmetrically, does not fit much better. When inspecting the issue points in the Unfolding analysis it became clear that all of them are located at the outskirts of, or outside the target configuration. This implies that the regression weight for the adequate term is small. Besides, ABORTION, ORDER, PARTICIP, and ARMIES turned out to be mapped as ideal-points. These are easy to interpret in this context, since we only have to reverse the interpretation of a small value on the issue scales.

Nevertheless, the combination of the just slightly better fit and the small (negative) regression weights for the quadratic term leads us to prefer the vec-

[1]The data of the Parliament Survey were collected by a team of political scientists of the Department of Political Science at Leiden University. The project was supported in part by the Netherlands Organization for the Advancement of Pure Research (ZWO), under grants 43-03 and 43-09.

Table 8.4: Political issues in the Parliament 1972 study.

Issue	lower end (1)	(9) upper end
1. DEVELOPMENT *AID*	Government should spend *less* money on aid to developing countries	Government should spend *more* money on aid to developing countries
2. *ABORTION*	A *woman has the right* to decide for herself about abortion	Government should *prohibit* abortion completely
3. LAW & *ORDER*	Government should take *stronger* action against public disturbances	Government takes *too strong* action against public disturbances
4. *INCOME* DIFFERENCES	Income differences should become *much less*	Income differences should *remain* as they are
5. *PARTICIP*ATION	*Workers too* must have participation in decisions important in industry	*Only management* should decide important matters in industry
6. *TAXES*	Taxes should be *decreased* so that people can decide for themselves	Taxes should be *increased* for general welfare
7. *ARMIES*	Government should insist on *maintaining* strong Western armies	Government should insist on *shrinking* the Western armies

Table 8.5: Fit measures for Parliament 1972.

Issue	VM	VP	UM	UP
AID	.692	.845	.714	.870
ABORTION	.609	.836	.665	.839
ORDER	.793	.925	.794	.930
INCOME	.703	.867	.710	.876
PARTICIP	.654	.875	.727	.877
TAXES	.791	.902	.813	.908
ARMIES	.772	.872	.772	.880

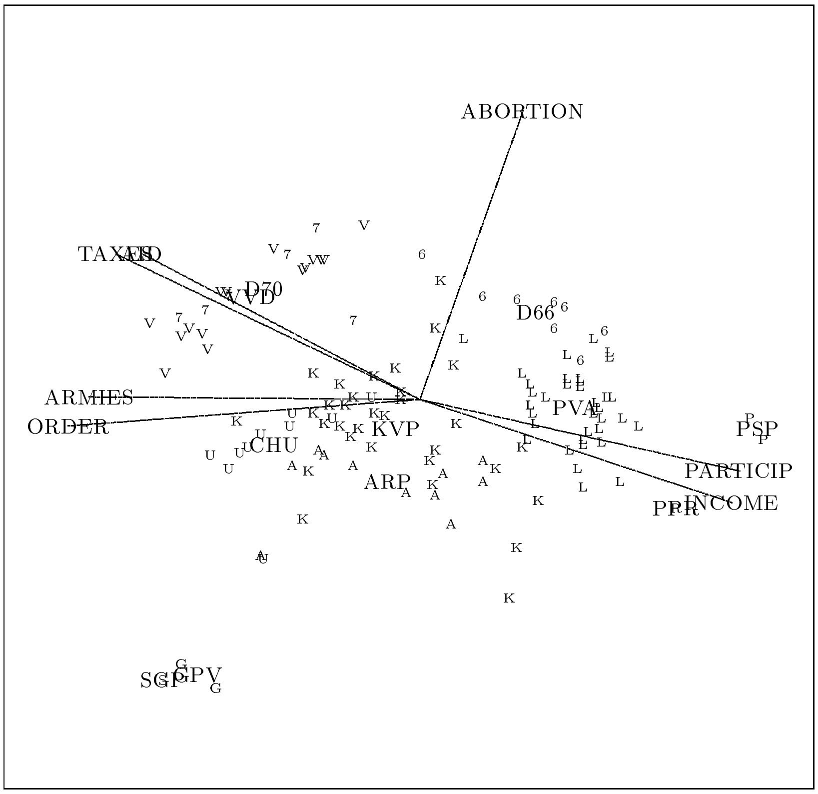

Figure 8.2: Joint plot of the MPs, parties and issues for the Parliament 1972 study.

tor model over the Unfolding model. We see the joint plot in Figure 8.2. Apart from the target points (MP's, labeled with small capitals) the column points from the SMACOF3 internal Unfolding analysis are plotted as well (the political parties, labeled with large capitals). The latter have not been used in the PREFMAP-3 analysis, but are given for completeness; they are in fact located at the centroid of the MPs belonging to that party (a restriction used in the internal Unfolding analysis). The vectors should be interpreted according to the lower end descriptions of the issue scales. The horizontal axis clearly reflects the left-right distinction in Dutch politics (plotted in reversed order), coherent with issues like income differences ("much less" versus "remain"), armies ("shrinking" versus "maintaining"), participation ("workers too" versus "management only"), and law & order ("too strong" versus "stronger"). But the second dimension should not be disregarded. Here the abortion issue brings together the left-wing parties (PSP, PVA, D66) and the (economically conservative) liberal parties D70 and VVD. They are opposed to the denominational parties (KVP, ARP, CHU, SGP, GPV) and, although to a lesser extent, to the left wing party PPR. The presence of this latter party on the "prohibit" side of the abortion vector can be easily explained by the fact that this MP used to belong to the catholic party KVP.

On the whole, the representation of MPs, parties and issues is quite con-

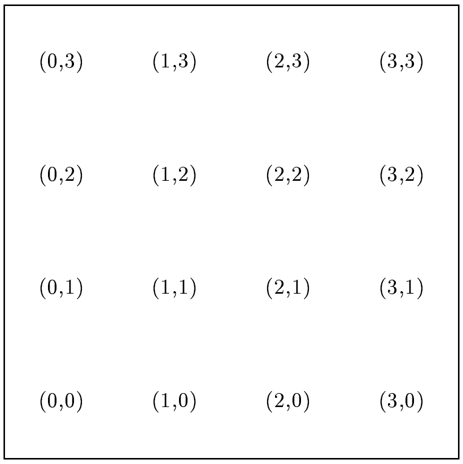

Figure 8.3: Target configuration for the analysis of family composition preference.

vincing, because it accumulates evidence against the idea of a one-dimensional polity on the one hand, and, on the other hand, it demonstrates the apparent possibility of predicting the stands on seven major political issues from a two-dimensional preference space. It summarizes, with little loss of information, 2232 observations in 284 parameters. The close resemblance to Heiser's (1981) analysis is also worth mentioning.

8.3 Preference for family composition

Our final application is a partial reanalysis of preference data collected by Delbeke (1978)[2]. The objects in this study are 16 family types, the individuals 82 undergraduates at Leuven University in Belgium, who ranked the family types in order of preference. Family types are defined as all combinations of *number of sons* and *number of daughters*, each ranging from 0 to 3. Thus (2,1) indicates two sons and one daughter, and (0,3) means no sons and three daughters. The number of sons and the number of daughters have been used to create two variables, defining the target configuration (see Figure 8.3). When we would connect the points horizontally, we find family types with an equal number of daughters. Connecting the points vertically gives families with an equal number of boys.

In Coombs' (*et al.*, 1973) theory regarding family composition preferences two new variables are defined in terms of the old ones. These are the *size* and the *sex* of the families. The main idea of the present analysis is to apply PREFMAP-3 with a *preliminary full transformation* of the target configuration. This might give us a clue on the question which of the two possibilities is more appropriate for describing Delbeke's family composition preferences.

The average fit for the 82 subjects, analyzing their preferences with the Unfolding model, metrically, with preliminary full transformation, is .906; the proportion of the total variance accounted for is .823. Without a preliminary

[2]We are indebted to Luc Delbeke for kindly making these data available to us.

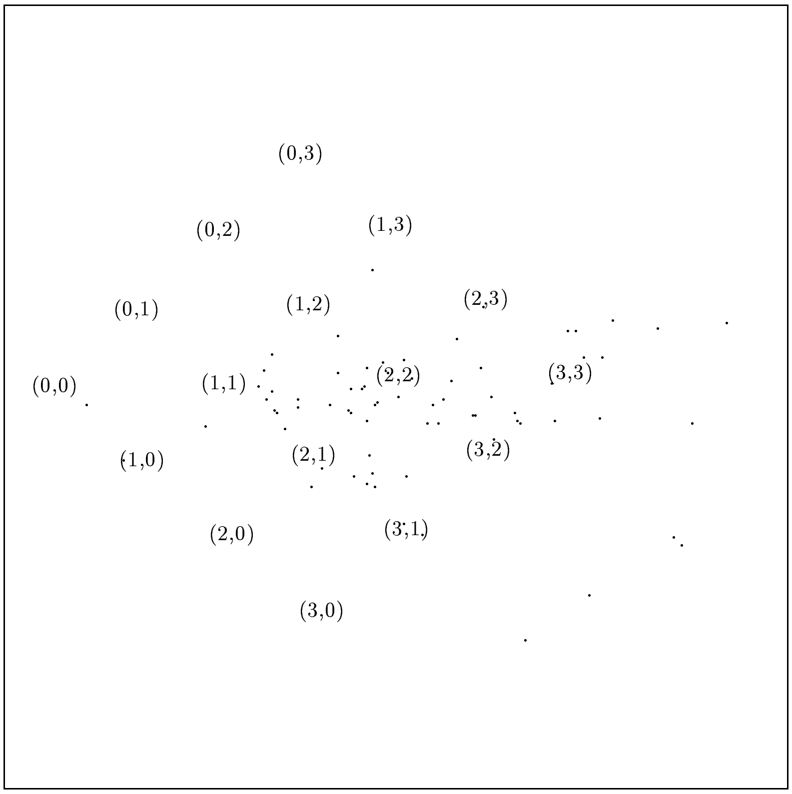

Figure 8.4: Transformed target configuration and ideal points from the analysis of preferences for family composition.

full transformation, these figures are .899 and .814, resp. The increase in fit is obviously not dramatic. For the preliminary transformation weights of 1.060 and .936 were obtained, for the first new axis and the second, resp. Thus the axes are only slightly differentially weighted. But the new axes' orientation is interesting (see Figure 8.4). Here the subjects points are labeled with a "x" (nine individuals are not included in the plot because their coordinates were severely out of range). It is clear that the transformation recovered the "size/sex" set of variables. Looking at the family points we now have a large bias towards daughters at the top, moving to a large bias towards sons at the bottom. The horizontal axis coincides with the total number of children. Evidently, this sample of Belgium students has high preference for large families, while sons are preferred over daughters.

For further analyses on this kind of data, we refer to Coxon (1974), Bechtel (1976), and Rodgers and Young (1981).

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Appendix A

Comparison with PREFMAP-2 and implementation

Only the most important differences between the PREFMAP-2 and the PREFMAP-3 program will be described here; there are many disparities in details, most notably in the input and the output, but these will be evident to anyone switching from one program to the other.

From a practical point of view the PREFMAP-3 program has been designed to be as *flexible* as possible. The array allocation is done dynamically, so that – given a large enough array to start with – there are no severe limitations to the number of individuals, objects, or dimensions to be analyzed (see Table A.1 for examples). It is even possible to analyze an "*infinite*" number of individuals without any increase in space beyond what is required to analyze one single individual. In that case the fact that every row of the external data matrix is analyzed separately is maximally exploited. Although we don't yet have experience with it, we expect this feature will make it feasible to run PREFMAP-3 smoothly on a Personal Computer (or other machines with modest core memory). Note, however, that it will still require space to obtain print output dealing with all individuals simultaneously (e.g., plotting the ideal points in the target configuration); thus one should refrain from those specifications when the data matrix is very large (and array area is limited). However, the individual results will never be completely lost, since they can be routed to other output units to provide for the possibility of constructing composite plots afterwards. Specific suggestions for dealing with various kinds of situations are given in section 6.5.

A second major difference with the PREFMAP-2 program consists in the choice of the *organization of the model fitting process*. In the PREFMAP-2 program the same model must necessarily be fitted, at each phase of the analysis, to each row of the data matrix. One has to start at one stage in the hierarchy and to stop at another, solving for all intermediate models. Moreover, the choice of the starting point affects the results for the simpler models in the PREFMAP-2 program. Those organizational features have been changed quite radically in the present program. Any ordered combination of models can be chosen, for example the most complex (the general ideal point model) and the

Table A.1: Examples of analyses that can be performed within a specified superarray size of 25100.

NROW	NCOL	NDIM	NANA	IPRE	VP	UP	WP	GP	IPRI	IPLO	NWORDS
350	20	3	4	2	1	1	1	1	1	3	24033
500	65	2	4	2	1	1	1	1	1	2	24934
1000	75	2	3	2	1	1	1	0	1	2	24307
1000	115	2	1	2	0	1	0	0	1	2	24831
2000	85	2	1	2	0	1	0	0	1	2	24643
3000	40	2	1	2	0	1	0	0	1	2	24263
3500	10	2	1	2	0	1	0	0	1	2	25093
99999	130	3	1	2	0	1	0	0	0	0	24266
99999	730	4	1	0	0	1	0	0	0	0	24934
99999	890	2	1	0	0	1	0	0	0	0	24794

Legend:

NROW	= number of rows	UP	= Unfolding model, primary app.
NCOL	= number of columns	WP	= weighted Unfolding model, ditto
NDIM	= number of dimensions	GP	= general Unfolding model, ditto
NANA	= number of analyses	IPRI	= print selected results
IPRE	= preliminary transformation	IPLO	= plot pairwise dimensions
VP	= vector model, primary approach	NWORDS	= total array area

simplest (the vector model), and they will be applied to the same (transformed) target configuration. In addition, the separate rows need not be fitted by the same models, although that is still possible. The possibility exists to give each row its own set of models.

The final major difference between PREFMAP-2 and PREFMAP-3 that we want to mention consists in the fact that the present program performs only *purely external analyses*. In the PREFMAP-2 program one out of a number of possible object configurations could be generated. This object configuration was obtained from the very same data matrix, whereas the rows would be fitted in afterwards in the second stage of the analysis. This two-stage process in fact renders an internal Unfolding solution. Since there are many general MDS and PCA programs available that perform internal analyses under various criteria, this possibility does not exist anymore in PREFMAP-3. If the user desires to employ, in PREFMAP-3, an object configuration derived from the external data, this configuration must be calculated with some auxiliary procedure, and must then be inserted in the input stream to the PREFMAP-3 program.

Implementation

PREFMAP-3 is a portable ANSI FORTRAN-IV program. It has been successfully tested on IBM, Honeywell, and VAX computers. The program contains a FORTRAN array allocation routine, called DECLAR. This routine allocates a superarray of a certain specified length.

The required size of the superarray depends on a number of parameters, some of which vary with the size of the problem (number of objects, dimensions), while others depend on the user-selected analysis and print specifications. If the superarray is not large enough, the program will return from DECLAR with an error message and with the correct size of the superarray. Table A.1 gives an overview of different analyses that can be performed within the internally fixed upper bound of the superarray (25100 words). This upper bound can, of course, be enlarged when implementing the program. If a genuine dynamic storage allocation facility is derived, subroutine DECLAR should be replaced by a machine assembler routine.

On some installations the plots produced by the program might come out rectangular instead of square. Several statements in the subroutine PRPLOT should be adapted according to the comments in this subroutine. The logical unit numbers to read the standard input stream and to write the standard output stream are defined in the main routine by the statements INPARA = 5 and IWRITE = 6, respectively.

Program structure

In terms of its subroutines, the program is structured as indicated in Figure A.1. Here, successive subroutine calls are given from top to bottom, whereas further calls into a deeper level are depicted horizontally. The subroutine MAIN3 controls the flow of the program. PREREC is used to perform printing of the input data and configuration, PREPO performs the preliminary transformation, PRED1 up to PRED4 construct predictor matrices suitable for the selected models, and PSINV computes their pseudo inverses.

The program contains two major loops. The inner one is across options, the outer one is across rows of the data matrix. The inner loop computes for each option successively:

(1) the predicted values for the metric case (ESTIMA);

(2) the optimal monotone function for the nonmetric case (by cycling between PROJEC and MONOR, the Alternating Least Squares iterations);

(3) the predicted values for the nonmetric case (ESTIMA);

(4) the parameter estimates of the original model (UNRA1 up to UNRA4);

(5) RESUL1 and RESUL2 take care of the various output options.

When no composite plotting or printing is asked for, the individual results will be lost from here on. After the loop across options is finished, statistics across options are given (STATIS). Then the program continues by finishing the outer loop across individuals. Composite results are provided for by OUTP3. Of course, the program skips subroutines when the activities performed by them are not called for by the user.

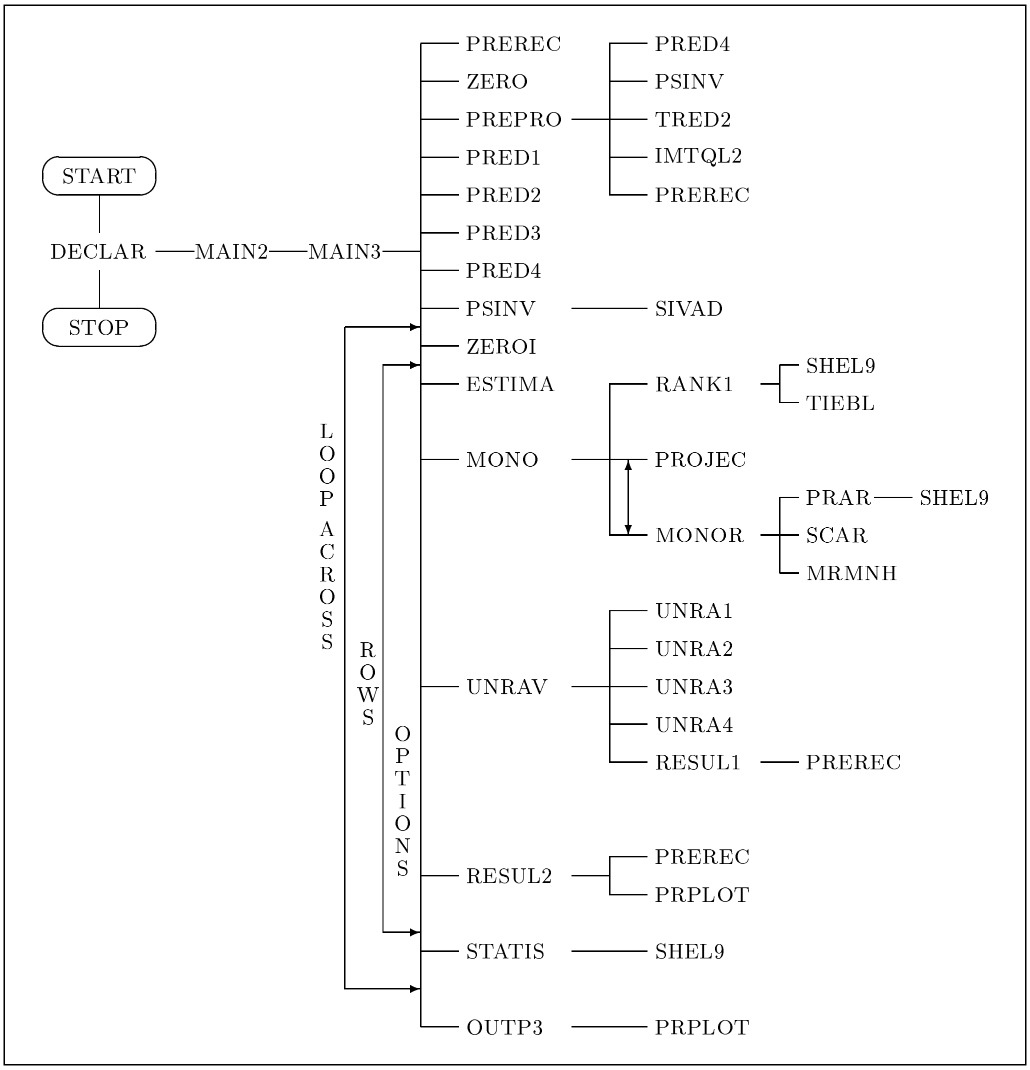

Figure A.1: Flow of the PREFMAP-3 program.

Appendix B

Deck set-up PREFMAP-3

CARD 1: TITLE CARD (section 6.1)

Col.	Format	Information
1–80	20A4	Any alphanumeric information to title the printout

CARD 2: DATA SPECIFICATION (section 6.3)

Col.	Format	Information
1–5	I5	number of row points
6–10	I5	number of column points
11–15	I5	option set selection 0 = all rows same option set 1 = all rows different option set 2 = specified rows same option set
16–20	I5	row number to start first option set (only needed if col. 11–15 equals 2; ditto for the next three parameters)
21–25	I5	row number to start second option set
26–30	I5	row number to start third option set
31–35	I5	row number to start fourth option set

CARD 3: ANALYSIS SPECIFICATION (section 6.4)

Col.	Format	Information
1–5	I5	number of dimensions
6–10	I5	maximum number of analyses in any option set
11–15	I5	preliminary transformation of configuration 0 = remains unchanged 1 = weighted 2 = rotated and weighted
16–20	I5	standardize external data 0 = yes (default, amounts to 1 + 2) 1 = center only 2 = normalize only 3 = none of the above
21–25	I5	application vector model (in any option set) 0 = no 1 = yes
26–30	I5	application Unfolding model (in any option set) 0 = no 1 = yes
31–35	I5	application weighted Unfolding model (in any option set) 0 = no 1 = yes
36–40	I5	application general Unfolding model (in any option set) 0 = no 1 = yes
41–45	I5	number of nonmetric iterations (default = 50)
46–55	F10.8	convergence criterion for nonmetric regression (default = .10E-04)

CARD 4: PRINT/PLOT OPTIONS (section 6.5)

Col.	Format	Information
1–5	I5	print input 0 = 10 rows of the external data at most 1 = target configuration 2 = external data 3 = target configuration as well as external data
6–10	I5	print results for each option 0 = none 1 = complete results 2 = fit only 3 = fit + criterion and predicted values
11–15	I5	print selected results for each analysis 0 = none 1 = coordinates, weights, rotation matrices
16–20	I5	scatter and transformation plots 0 = no plots N = plot for first N rows
21–25	I5	plot ideal points in target configuration 0 = no plots 1 = plot first dimension only 2 = plot first two dimensions K = plot K dimensions (pairwise)
26–30	I5	print history of nonmetric regression 0 = no print 1 = print for each row
31–35	I5	print F-statistic for metric analyses 0 = no statistics 1 = F-statistic for each row
36–40	I5	store output 1 0 = no storage 1 = store coordinates 2 = store coordinates and weights 3 = store coordinates, weights and rotation matrices
41–45	I5	store output 2 0 = no storage 1 = store predicted values 2 = store predicted values and criterion values

CARD 5: UNIT NUMBERS FOR INPUT/OUTPUT (section 6.6)

Col.	Format	Information
1–5	I5	unit number to read the target configuration from (default = 5)
6–10	I5	unit number to read the external data from (default = 5)
11–15	I5	unit number to read the option table (default = 5)
16–20	I5	unit number for scratch file 1
21–25	I5	unit number for scratch file 2
26–30	I5	unit number to store the coordinates (default = 6)
31–35	I5	unit number to store the weights (default = 6)
36–40	I5	unit number to store the rotation matrices (default = 6)
41–45	I5	unit number to store the predicted and the criterion values (default = 6)

CARD 6: FORMAT CARD FOR THE CONFIGURATION (section 6.1)

Col.	Format	Information
1–80	20A4	FORTRAN F-format to read the configuration

TARGET CONFIGURATION CARDS (section 6.1)
The target configuration must appear here if the input medium is unit 5. The configuration must have the form as given in Figure 1.2, i.e. as many (groups of) cards as there are objects, and as many fields as there are dimensions

FORMAT CARD FOR THE EXTERNAL DATA (section 6.1)

Col.	Format	Information
1–80	20A4	FORTRAN F-format to read the external data

EXTERNAL DATA MATRIX CARDS (section 6.1)
The external data must appear here if the input medium is unit 5. The data matrix must have the form as given in Figure 1.1, i.e. as many (groups of) cards as there are individuals, and as many fields as there are objects

OPTION TABLE CARDS (section 6.2)

Col.	Format	Information
1–16	4A4	The option table must appear here if the input medium is unit 5. Each card contains one option set, consisting of 1 up to 4 options. Each option is coded with two characters; the first option code starts in column 1, the second starts in column 5, etc.

First characters may be:

V=Vector model
U=Unfolding model
W=Weighted Unfolding model
W=General Unfolding model

Second characters may be:

M=Metric regression
P=Nonmetric regression, primary approach to ties
S=Nonmetric regression, secondary approach to ties